HW #3

4300:704 Finite Element Analysis II

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SPRING SEMESTER

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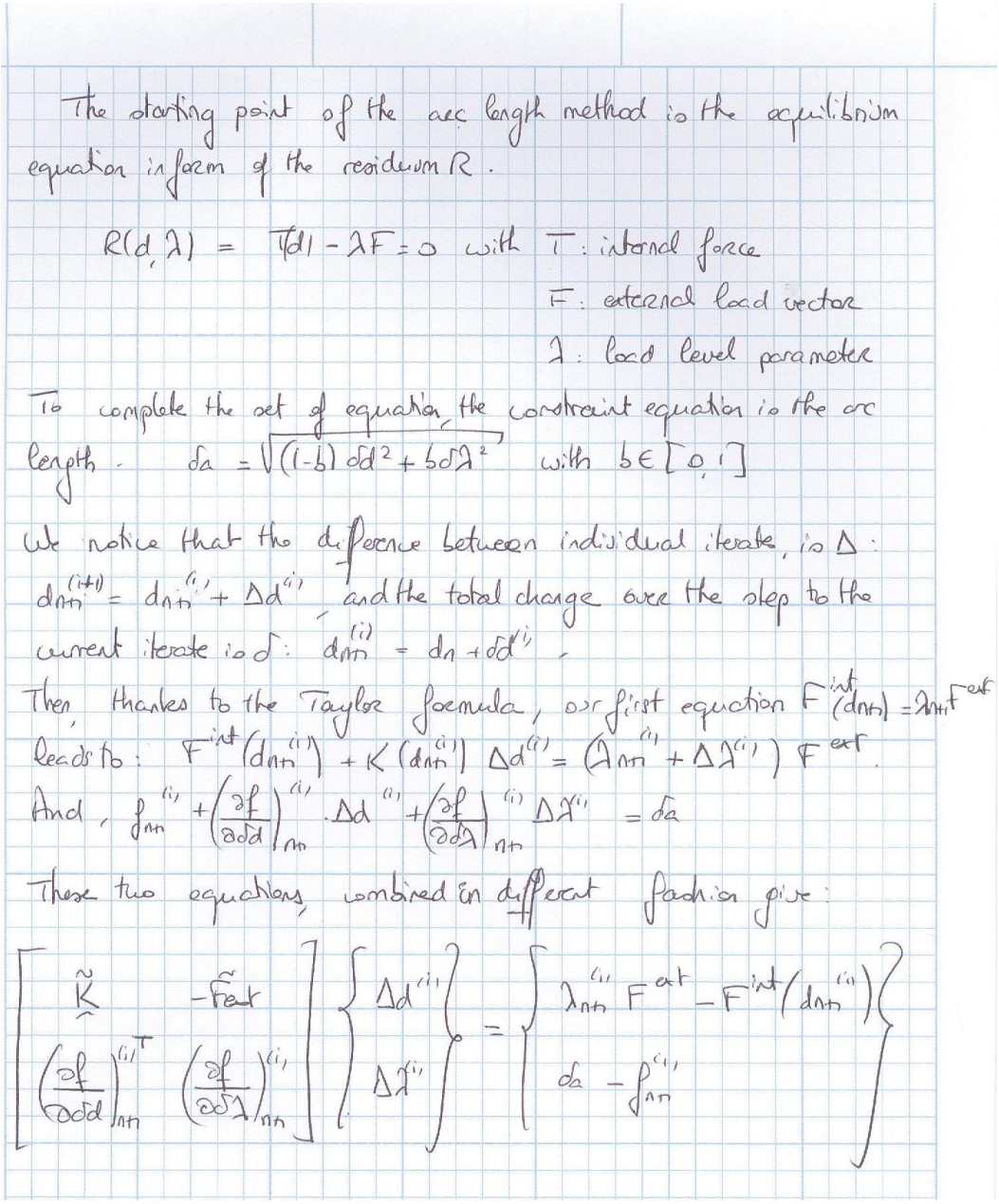
Consider the relation $F = 0.2d^3 - 2.1d^2 + 6d$.

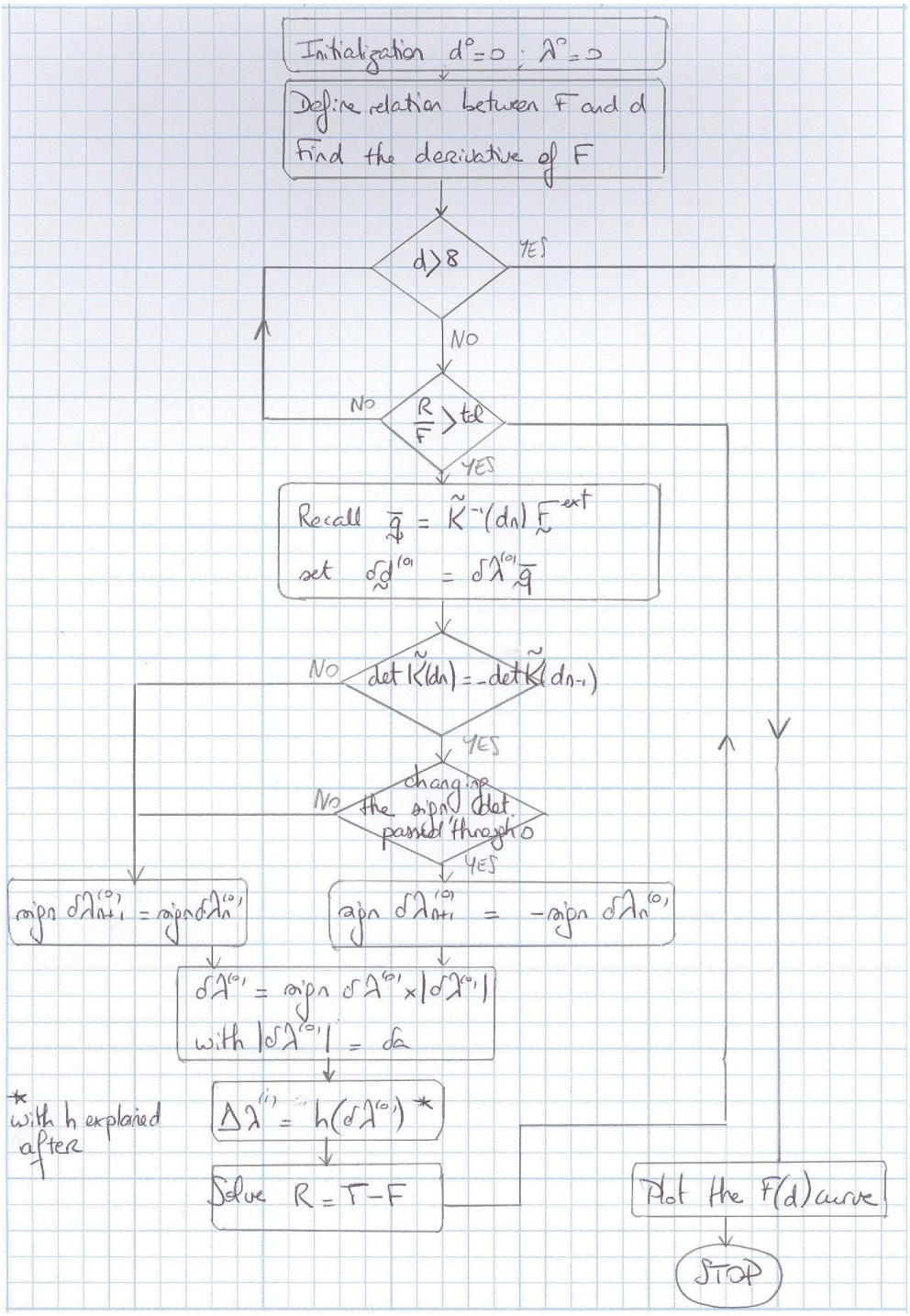
The aim of this homework is to use the Arc-length control with b = ½ in steps of δa =1.0 to obtain the nonlinear curve in the range of $d \in [0.0,8.0]$ by the Modified Newton-Raphson method that we studied in the previous homework.

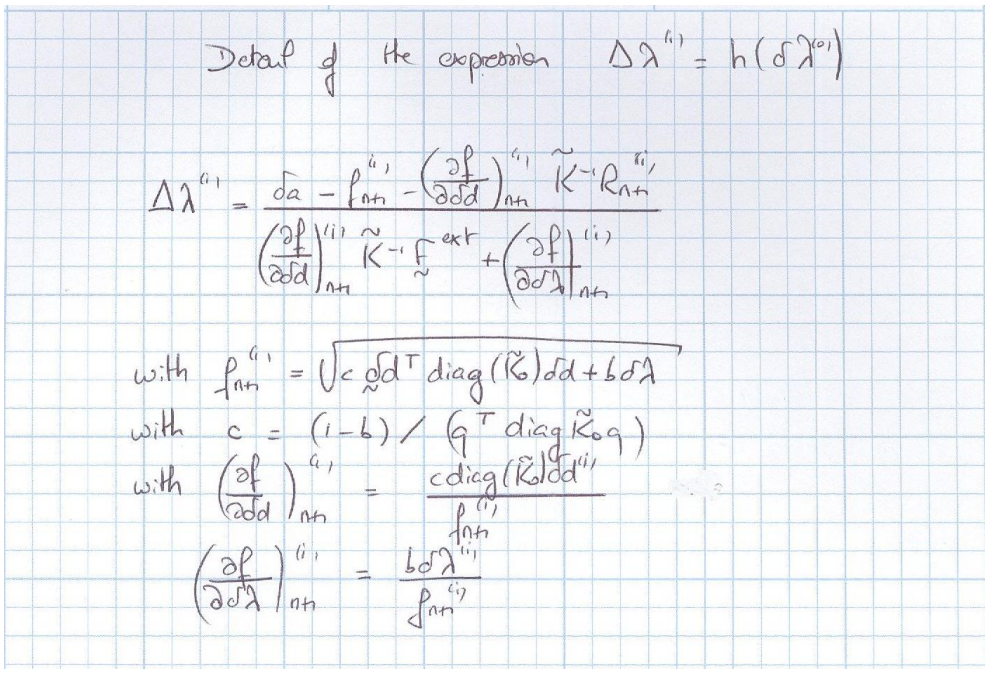
We'll still use the tolerance parameter $\varepsilon=10^{-12}$

I. Algorithm and solution procedure



II. Flow chart





III. Program

```
% Finite Element  Analysis II Hwk#3
% Obtain the nonlinear curve in the range of d(0.0,8.0)with
% F=0.2d^2-2.1d^3+6d using Modified Newton-Raphson method
clc
clear all

da=1;
b=1/2;
e = 10^-12; %Tolerance parameter
imax=500;
d(1)=0;
F(1)=0;
n=1;

%Arc length method set
dlanda(2,1)=da;%deltalanda
dlanda(1,1)=da;%dlanda


while(d(n)<=8)
    n
    Fext(n)=1;
    fext(n)=1;
    landa(n,1)=F(n);
    K(n)=0.6*d(n,1)^2-4.2*d(n,1)+6;

    if n>=3 && K(n)*K(n-1)<0
        dlanda(n+1,1)=-dlanda(n,1);
    else
        dlanda(n+1,1)=dlanda(n,1);
```

```
    end
    q(n)=K(n)^-1*fext(n);

    if abs(q(n))>da
        q(n)=q(n-1);
    end
    dd(n+1,1)=abs(dlanda(n+1,1)*q(n));
    d(n+1,1)=d(n,1)+dd(n+1,1);

    landa(n+1,1)=landa(n,1)+dlanda(n+1,1);

    i=1;
    iloop=1;

    while (i<imax)&&(iloop>0)
        f(n+1,i)=(0.5*dd(n+1,i)^2/q(n)^2+0.5*dlanda(n+1,i)^2)^0.5;
        DfDlanda(n+1,i)=b/f(n+1,i)*dlanda(n+1,i);
        c=(1-b)/(K(n)*q(n)^2);
        DfDd(n+1,i)=c/f(n+1,i)*K(n)*dd(n+1,i);
        Fint(n+1,i)=0.2*d(n+1,i)^3-2.1*d(n+1,i)^2+6*d(n+1,i);
        R(n+1,i)=landa(n+1,i)*Fext(n)-Fint(n+1,i);

        %Definition of the left hand side matrix
        A(1,1)=K(n);
        A(1,2)=-Fext(n);
        A(2,1)=DfDd(n+1,i);
        A(2,2)=DfDlanda(n+1,i);
        ARem{n+1,i}=A;

        %Definition of the right hand side matrix
        B(1,1)=R(n+1,i);
        B(2,1)=da-f(n+1,i);

        %Solution
        S=A^-1*B;
        Dd(n+1,i)=S(1,1);
        Dlanda(n+1,i)=S(2,1);

        d(n+1,i+1)=d(n+1,i)+Dd(n+1,i);
        landa(n+1,i+1)=landa(n+1,i)+Dlanda(n+1,i);
        dd(n+1,i+1)=d(n+1,i+1)-d(n,1);
        dlanda(n+1,i+1)=landa(n+1,i+1)-landa(n,1);
        fprintf('iteration = %3.0f R = %4.13f\n',i, R(n+1,i))
        if abs(R(n+1,i))<e*abs(R(n+1,1))
            iloop=-1;
        end
        i=i+1;

    end

    F(n+1) = landa(n+1,i-1)*Fext(n);
    d(n+1) = d(n+1,i-1);
    n=n+1;
end
```

```
%Plot the curve
plot(d(:,1), F)
xlabel('d')
ylabel('F')
title('Arc-lenght control with modified Newton-Raphson method')
```

IV. Computed results

We are attaching the results indicating the number of iterations required for each load step and the residual reduction in each iteration.

n = 1
iteration = 1 R = 0.0574074074074
iteration = 2 R = 0.0033147833916
iteration = 3 R = 0.0001403110297
iteration = 4 R = 0.0000078098234
iteration = 5 R = 0.0000004434950
iteration = 6 R = 0.0000000252001
iteration = 7 R = 0.0000000014320
iteration = 8 R = 0.0000000000814
iteration = 9 R = 0.0000000000046
iteration = 10 R = 0.0000000000003
iteration = 11 R = 0.0000000000000

n =2
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iteration = 4 R = 0.0000169313731
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iteration = 6 R = 0.0000000803928
iteration = 7 R = 0.0000000055425
iteration = 8 R = 0.0000000003821
iteration = 9 R = 0.0000000000263
iteration = 10 R = 0.0000000000018
iteration = 11 R = 0.0000000000001
iteration = 12 R = 0.0000000000000

n =3
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iteration = 4 R = 0.0000441900189
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iteration = 6 R = 0.0000003384785
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iteration = 11 R = 0.0000000000017
iteration = 12 R = 0.0000000000002
iteration = 13 R = 0.0000000000000

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iteration = 10 R = 0.0000000004592
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iteration = 13 R = 0.0000000000008
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iteration = 7 R = 0.0000061797669
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iteration = 10 R = 0.0000000405545
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iteration = 12 R = 0.0000000014216
iteration = 13 R = 0.0000000002662
iteration = 14 R = 0.0000000000498
iteration = 15 R = 0.0000000000093
iteration = 16 R = 0.0000000000017

iteration = 17 R = 0.0000000000003
iteration = 18 R = 0.0000000000001

n =6
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iteration = 2 R = 0.2426708297015
iteration = 3 R = 0.0693681642343
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iteration = 4 R = 0.0301863041076
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iteration = 14 R = 0.0000000000000

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iteration = 12 R = 0.0000000000001

n = 15

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iteration = 4 R = 0.0000032550121
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iteration = 2 R = 0.0015593671498
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iteration = 9 R = -0.0000000000000

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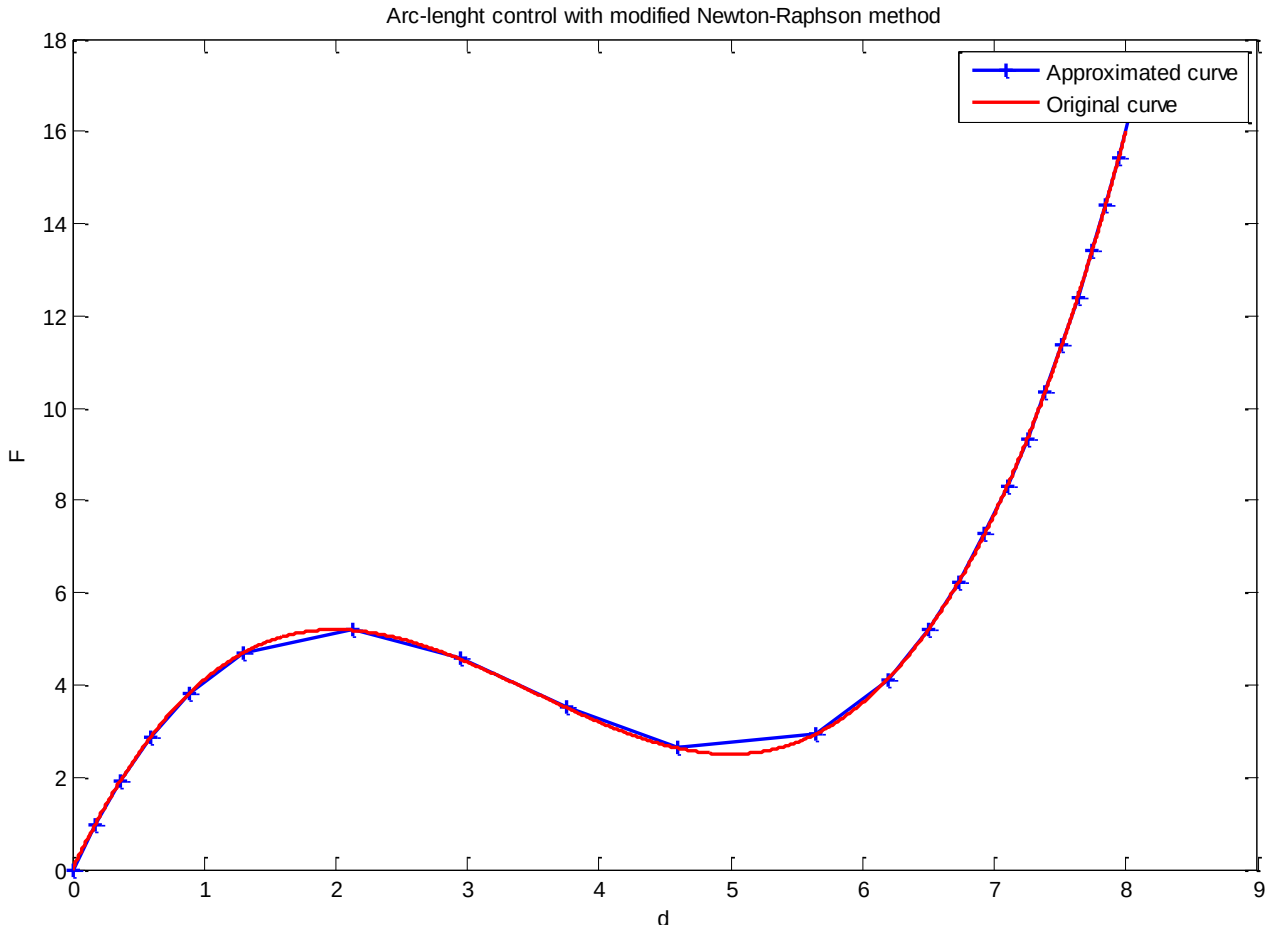
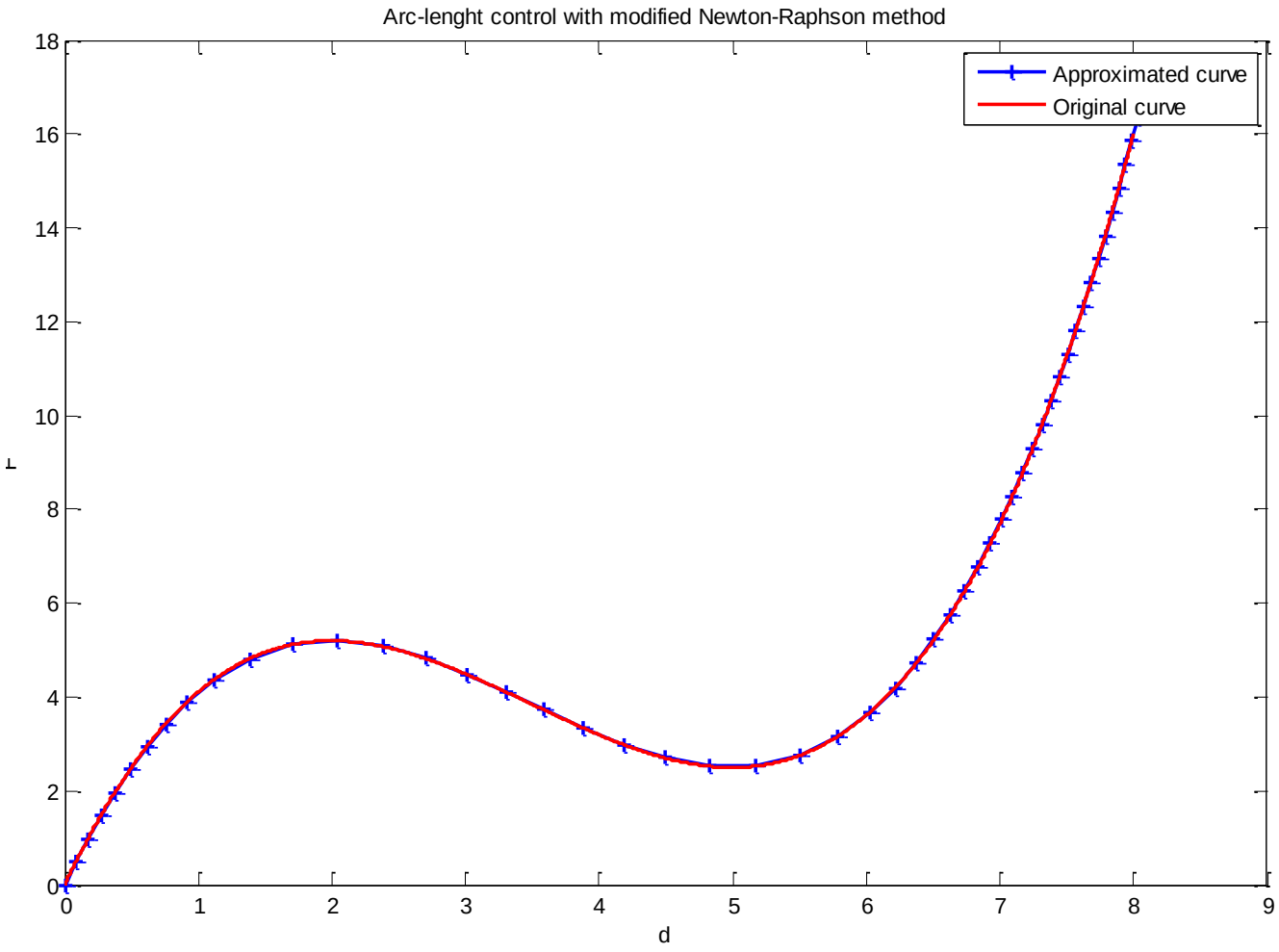
iteration = 9 R = -0.0000000000000

V. Force displacement curve



Let's reduce δa to ½ to get a higher resolution.



VI. Comments

The arc-length strategy allows plotting any complete nonlinear curve. Indeed, we used other methods previously (HW#2) and noticed that we couldn't plot after some limit point (d = 2.5). We applied the load monotonically without any knowledge of the nature of the nonlinear system.
So to find the whole path, we can't use a pure load control or a pure displacement control for the solution of nonlinear system.
Thus, arc-length method is a more effective solution, it is a continuation method. Instead of increasing $\delta\lambda$ of the load control, we increase the arc-length of the solution curve in load-displacement space δa. This measure is monotonic as the solution path evolves.
Moreover, the arc-length method improves the Modified Newton Raphson method in the way that it increase its speed. Without the arc-length constraint ($f=\delta a$), the modified Newton-Raphson iteration would converge more slowly. And it might not converge at all if the target load level exceeded some local maxima of the solution curve.