



Lecture 3: Digital Signal Processing for Analysis of Vibration Response

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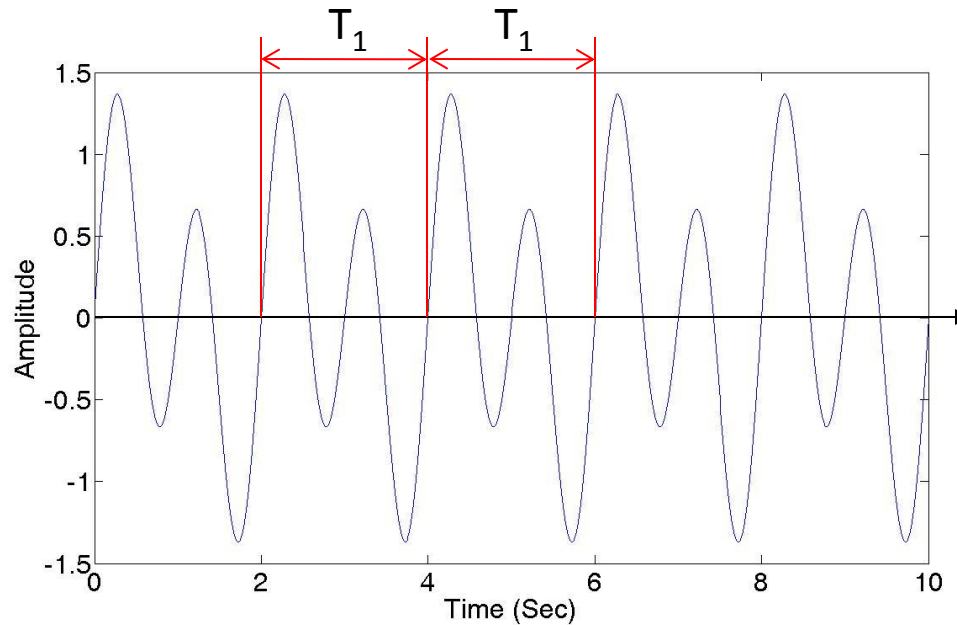
Learning Objectives

Learning Objectives

- ❑ Objective 1: Understand fundamental transforms for frequency-domain analysis of dynamic signals: Fourier Series, Fourier Integral Transform, Discrete Fourier Transform, Fast Fourier Transform, etc.
- ❑ Objective 2: Understand basics of digital signal processing

2

Fourier Series for Periodic Functions: Real and Complex Series



- The periodic function repeats itself indefinitely

$$p(t) = p(t + T_1) = p(t + 2T_1) = \dots$$

$$T_1 = \frac{2\pi}{\omega_1} \implies \omega_1 = \frac{2\pi}{T_1} \quad \text{Fundamental frequency}$$

- For any periodic function, a Fourier Series can be found as

$$p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\Omega_1 t) + \sum_{n=1}^{\infty} b_n \sin(n\Omega_1 t)$$

- The coefficients are as follows

$$a_0 = \frac{1}{T_1} \int_{\tau}^{\tau+T_1} p(t) dt \longrightarrow \text{Average of } p(t)$$

$$a_n = \frac{2}{T_1} \int_{\tau}^{\tau+T_1} p(t) \cos(n\Omega_1 t) dt \quad n=1,2,\dots$$

$$b_n = \frac{2}{T_1} \int_{\tau}^{\tau+T_1} p(t) \sin(n\Omega_1 t) dt \quad n=1,2,\dots$$

Real Fourier Series (2)

- Question) How do we use this representation to determine the solution for the system response?
- We know the steady-state response (amplitude and phase angle) for a SDOF system with harmonic input. By superposition we can find the response to the Fourier series representation of the input.

$$u(t) = u_0 + \underbrace{\sum_{n=1}^{\infty} u_n^c(t)}_{\text{cosine terms}} + \underbrace{\sum_{n=1}^{\infty} u_n^s(t)}_{\text{sine terms}}$$

$$u_0 = \frac{a_0}{k} \longrightarrow \text{Static deflection}$$

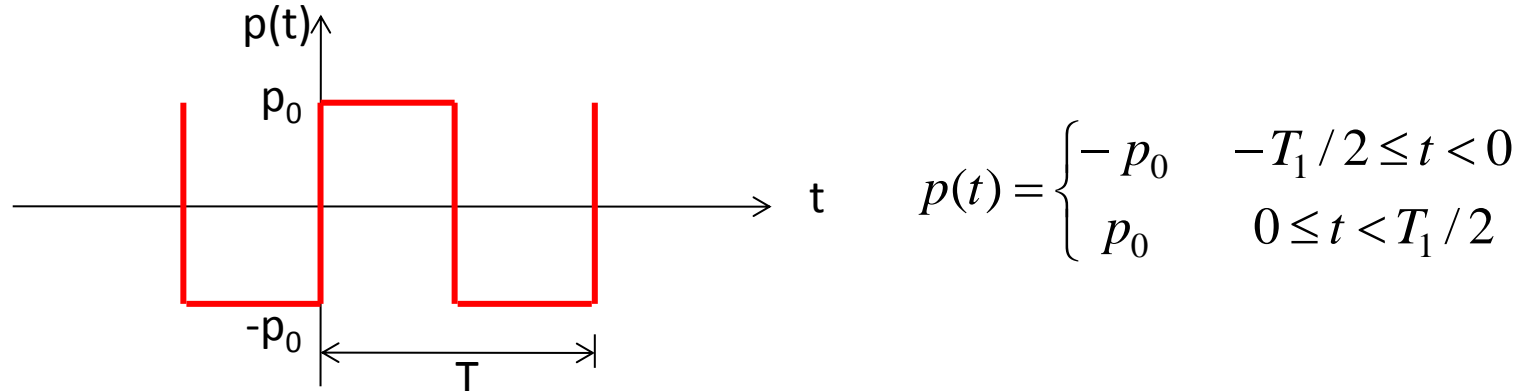
$$u_n^c(t) = \frac{a_n / k}{\sqrt{(1 - r_n^2)^2 + (2\zeta r_n)^2}} \cos(n\Omega_1 t - \alpha_n)$$

$$r_n = \frac{n\Omega_1}{\omega_n} \quad \text{Frequency ratio}$$

$$u_n^s(t) = \frac{a_n / k}{\sqrt{(1 - r_n^2)^2 + (2\zeta r_n)^2}} \cos(n\Omega_1 t - \alpha_n)$$

Example-1 (1)

- Determine a real Fourier series representation of a square wave



- Since $p(t)$ is a odd function, coefficients of cosine terms ($a_0=a_n=0$) will be zero.

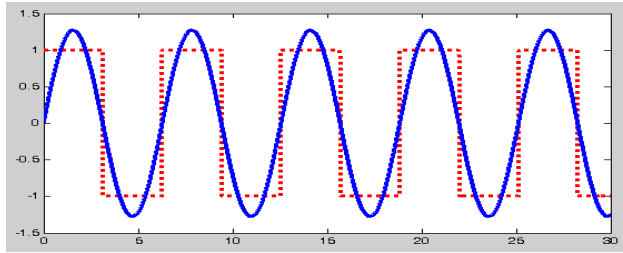
$$b_n = \frac{4p_0}{T_1} \int_0^{T_1/2} \sin(n\Omega_1 t) dt = \frac{4p_0}{n\pi} \quad n = 1, 3, 5, \dots$$

$$p(t) = \frac{4p_0}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \sin(n\Omega_1 t) \quad \Omega_1 = \frac{2\pi}{T_1}$$

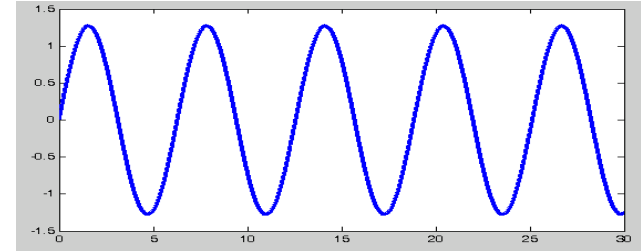
Example-1 (2)

□ When $T_1=2\pi$ and $p_0=1$

$$p(t) = \frac{4}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \sin(nt) = \sum_{n=1,3,5,\dots} p_n(t)$$

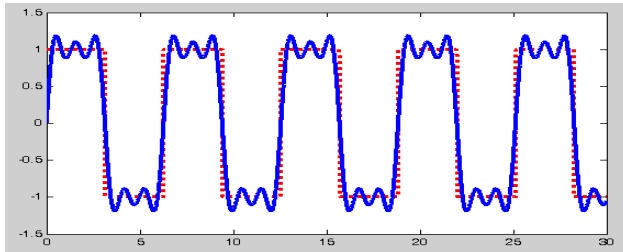


p_1

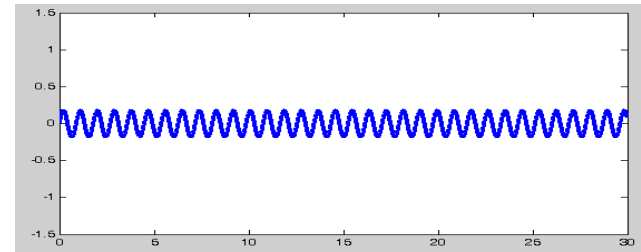


p_1

...

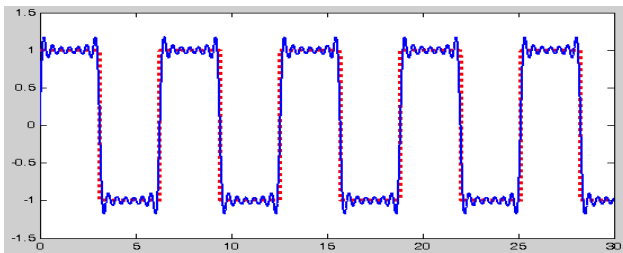


$p_1 + p_3 + p_5$

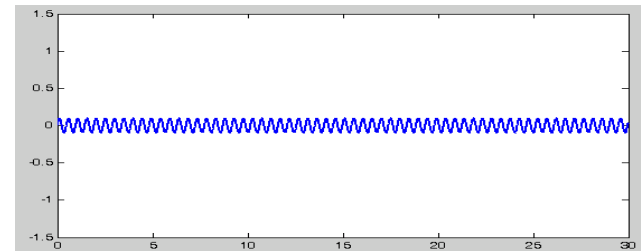


p_7

...



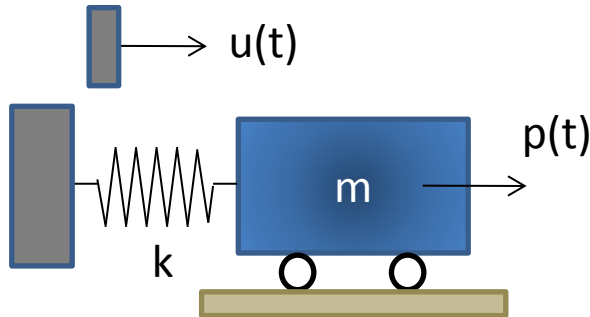
$p_1 + p_3 + p_5 + p_7 + p_9 + p_{11} + p_{13}$



p_{13}

Example-2 (1)

- Determine a Fourier series expression for the steady-state response of an undamped SDOF system subjected to the previous square-wave excitation $p(t)$. $\omega_n = 6\Omega_1$.



- Recalling the steady state response

$$u_p(t) = \frac{U_0}{1-r^2} \sin(\Omega t) \quad \text{subjected to } p(t) = p_0 \sin(\Omega t) \quad U_0 = p_0 / k$$

$$p(t) = \sum_{n=1,3,5,\dots} \frac{4p_0}{n\pi} \sin(n\Omega_1 t) = \sum_{n=1,3,5,\dots} p_n \sin(n\Omega_1 t)$$

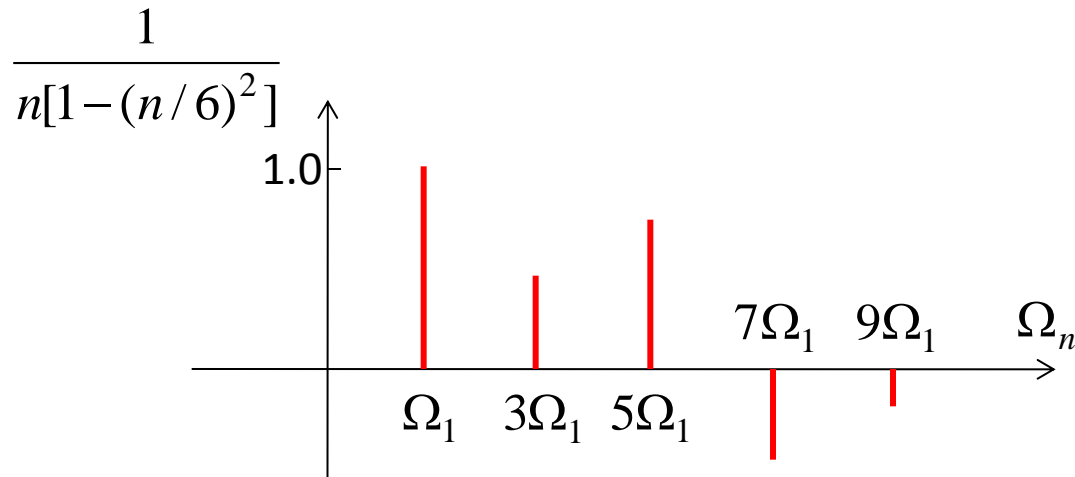
$$u(t) = \sum_{n=1,3,5,\dots} \frac{p_n / k}{1-r_n^2} \sin(n\Omega_1 t)$$

Example-2 (2)


□ As $r_n = \frac{n\Omega_1}{\omega_n} = \frac{n\Omega_1}{6\Omega_1} = \frac{n}{6}$,

$$u(t) = \frac{4p_0}{k\pi} \sum_{n=1,3,5,\dots} \frac{1}{n[1-(n/6)^2]} \sin(n\Omega_1 t)$$

□ We can graph the spectra of the response amplitude as follows



- The complex Fourier series takes the form

real 

$$p(t) = \sum_{n=-\infty}^{\infty} \bar{P}_n(\Omega) e^{in\Omega_1 t}$$

- Note that the periodic function $p(t)$ is real, which is represented by including negative n . $n=0, \pm 1, \pm 2, \pm 3, \dots$
- The complex coefficient is

$$\bar{P}_n = \frac{1}{T_1} \int_{\tau}^{\tau+T_1} p(t) e^{-i(n\Omega_1 t)} dt \quad n = 0, \pm 1, \dots \quad \text{-----} \rightarrow (1)$$

- Note that

$$\bar{P}_{-n} = \bar{P}_n^* = \text{complex conjugate of } \bar{P}_n$$

$$\bar{P}_0 = \frac{1}{T_1} \int_{\tau}^{\tau+T_1} p(t) dt = \text{average value of } p(t)$$

Example 3 – (1)

- Determine an expression for the Fourier coefficients \overline{P}_n of the Complex Fourier Series representation for the square wave in Example 1

- Evaluating Equation (1),

$$\overline{P}_n = \frac{1}{T_1} \int_0^{T_1/2} p_0 e^{-i(n\Omega_1 t)} dt + \frac{1}{T_1} \int_{T_1/2}^{T_1} (-p_0) e^{-i(n\Omega_1 t)} dt$$

$$\overline{P}_n = \frac{-p_0}{in\Omega_1 T_1} \left[e^{-i(n\Omega_1 t)} \Big|_0^{T_1/2} - e^{-i(n\Omega_1 t)} \Big|_{T_1/2}^{T_1} \right]$$

- Note that $\Omega_1 T_1 = 2\pi$

$$e^{-i(n\Omega_1 T_1/2)} = e^{-i(n\pi)} = \begin{cases} +1 & n = \text{even} \\ -1 & n = \text{odd} \end{cases}$$

$$e^{-i(n\Omega_1 T_1)} = e^{-i(2n\pi)} = 1$$

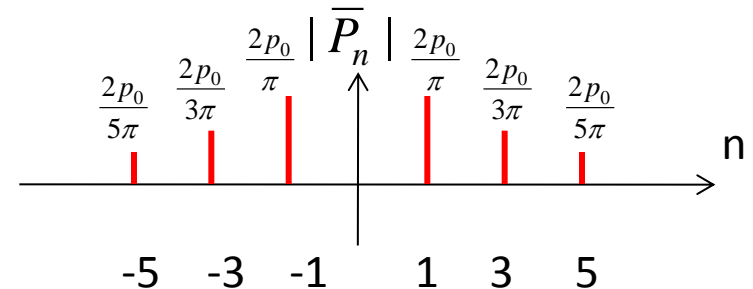
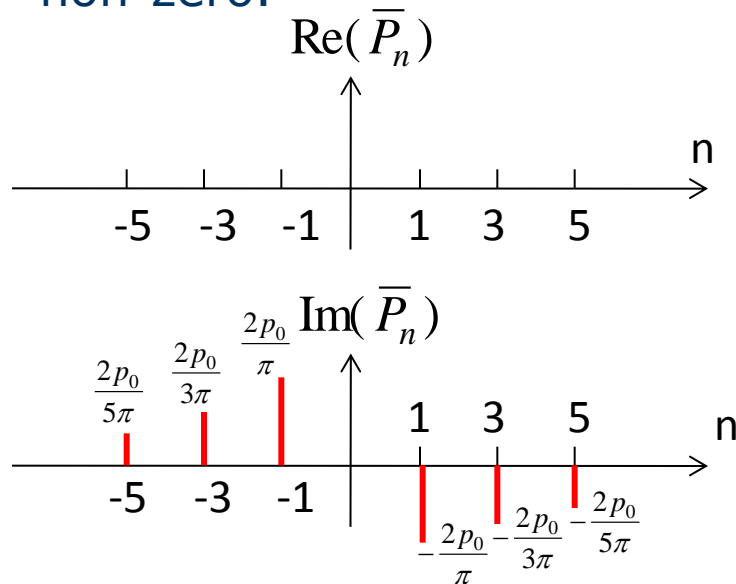
Example 3 – (2)

- Then the Complex Fourier coefficients are given by

$$\bar{P}_n = \frac{1}{T_1} \int_0^{T_1/2} p_0 e^{-i(n\Omega_1 t)} dt + \frac{1}{T_1} \int_{T_1/2}^{T_1} (-p_0) e^{-i(n\Omega_1 t)} dt$$

$$\bar{P}_n = \frac{ip_0}{2n\pi} (2e^{-i(n\pi)} - 2) = \begin{cases} 0 & n = \text{even} \\ -\frac{2ip_0}{n\pi} & n = \text{odd} \end{cases}$$

- Plotting its spectra, real parts are zero and only imaginary parts are non-zero.



3

Fourier Integral Transform for Non-periodic Functions

- When the function to be represented is not periodic, it can be represented by a Fourier integral
- From complex Fourier series, we know that

$$p(t) = \sum_{n=-\infty}^{\infty} \bar{P}_n e^{in\Omega_1 t} \quad \text{----->} \quad (2)$$

$$\bar{P}_n = \frac{1}{T_1} \int_{\tau}^{\tau+T_1} p(t) e^{-i(n\Omega_1 t)} dt \quad n = 0, \pm 1, \dots$$

- We can obtain the expression for Fourier transform by letting $T_1 \rightarrow \infty$ and defining $\Delta\Omega = \Omega_1$ and $\Omega_n = n\Omega_1$.

$$\bar{P}_n(\Omega_n) = T_1 \bar{P}_n = \frac{2\pi}{\Delta\Omega} \bar{P}_n$$

- Then Equation (2) can be rewritten as

$$p(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \bar{P}_n(\Omega) e^{i\Omega_n t} \Delta\Omega \qquad \bar{P}(\Omega_n) = \int_{-T_1/2}^{T_1/2} p(t) e^{-i(\Omega_n t)} dt$$

- As $T_1 \rightarrow \infty$, $\Delta\Omega$ becomes $d\Omega$ and Ω_n becomes a continuous variable Ω .

$$p(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{P}(\Omega) e^{i\Omega t} d\Omega$$

$$\bar{P}(\Omega_n) = \int_{-\infty}^{\infty} p(t) e^{-i(\Omega_n t)} dt$$

- Also, the Fourier transform pair can be written more symmetrically in the form, $\Omega = 2\pi f$, $d\Omega = 2\pi df$

$$p(t) = \int_{-\infty}^{\infty} \bar{P}(f) e^{i(2\pi f)t} df$$

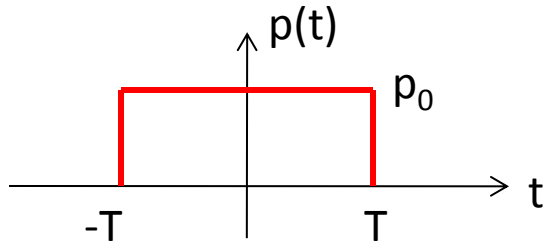
→ Inverse Fourier Transform

$$\bar{P}(f) = \int_{-\infty}^{\infty} p(t) e^{-i(2\pi f)t} dt$$

→ Fourier Transform

Example 4

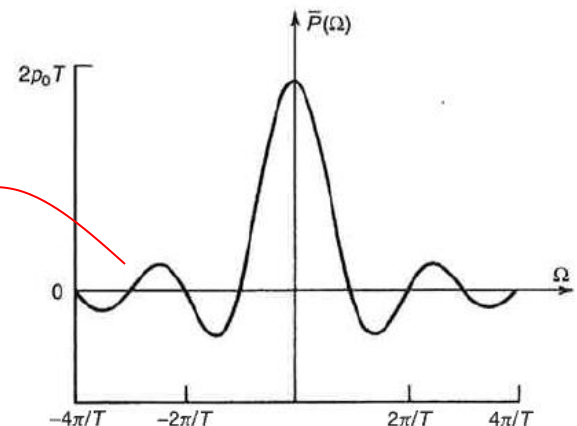
- Determine the Fourier transform of a rectangular pulse



$$\begin{aligned}\bar{P}(\Omega) &= \int_{-\infty}^{\infty} p(t) e^{-i(\Omega t)} dt = \int_{-T}^{T} p_0 e^{-i(\Omega t)} dt \\ &= -\frac{p_0}{i\Omega} \left(e^{-i\Omega t} \right) \Big|_{-T}^T = -\frac{p_0}{i\Omega} \left(e^{-i\Omega T} - e^{i\Omega T} \right) = \frac{2p_0}{\Omega} \sin(\Omega T)\end{aligned}$$

- Plotting this Fourier transform,

Ripple is due to
sharp corner



4

Discrete Fourier Transforms

Discrete Fourier Transform (1)

- ❑ Numerical computations of the Fourier Transform become a practical reality by development of Fast Fourier Transform (FFT) by Cooley-Tukey in 1965.

- ❑ Fourier Transform pair

$$p(t) = \int_{-\infty}^{\infty} \bar{P}(f) e^{i(2\pi f)t} df$$

$$\bar{P}(f) = \int_{-\infty}^{\infty} p(t) e^{-i(2\pi f)t} dt$$

- ❑ Finite Fourier Transform

$$\bar{P}(f, T) = \int_0^T p(t) e^{-i(2\pi f)t} dt$$



Still continuous sample of p(t)

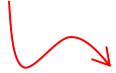
Discrete Fourier Transform (2)

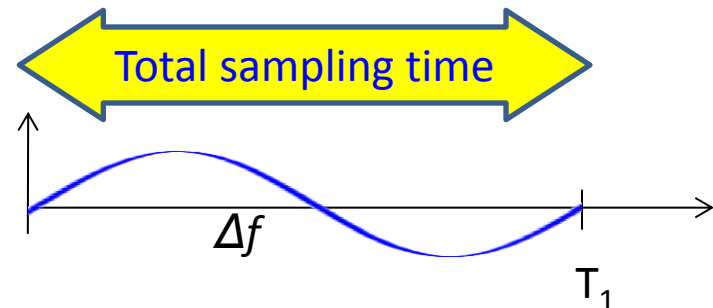
- Discrete Fourier Transform (DFT)
 - Sample $p(t)$ at N equally spaced points in time interval Δt $T_1 = N\Delta t$

$$p_m = p(m\Delta t) \quad m = 0, 1, 2, \dots, N-1$$

- Then the discrete version of finite Fourier Transform is

$$\bar{P}(f_n, T) = \Delta t \sum_{m=0}^{N-1} p(t_m) e^{-i(2\pi f_n)(m\Delta t)}$$

 function of f



- If the total sample time is T_1 , then the fundamental frequency sinusoid that fits within this sample time has a period T_1 . Therefore, the frequency interval of the discrete Fourier Transform is

$$\Delta f = \frac{1}{T_1} = \frac{1}{N\Delta t} \quad n\Delta f = \frac{n}{T_1} = \frac{n}{N\Delta t} \quad n = 0, 1, \dots, N-1$$

- Finally the Discrete Fourier Transform can be written as

$$\begin{aligned}\bar{P}(f_n) &= \Delta t \sum_{m=0}^{N-1} p(t_m) e^{-i2\pi(m\Delta t)(n\Delta f)} \\ &= \Delta t \sum_{m=0}^{N-1} p(t_m) e^{-i2\pi \frac{mn}{N}} \quad (n = 0, 1, \dots, N-1)\end{aligned}$$

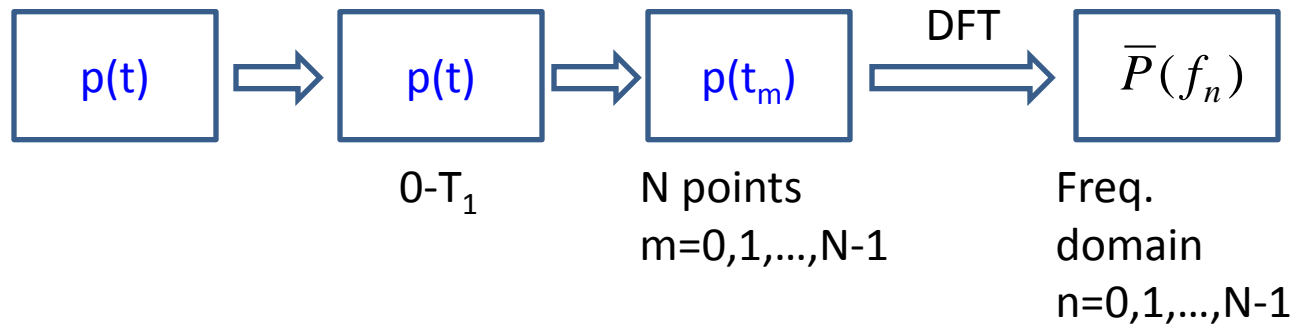
$\Delta f = \frac{1}{N\Delta t}$

- Then the inverse Discrete Fourier Transform can also be written as (from the integral equation)

$$\begin{aligned}p(t_m) &= \sum_{n=0}^{N-1} \bar{P}(f_n) e^{i2\pi(m\Delta t)(n\Delta f)} \\ &= \frac{1}{N\Delta t} \sum_{n=0}^{N-1} \bar{P}(f_n) e^{i2\pi \frac{mn}{N}} \quad (m = 0, 1, \dots, N-1)\end{aligned}$$

Δt is a scale factor

Discrete Fourier Transform (4)



5

Fast Fourier Transforms

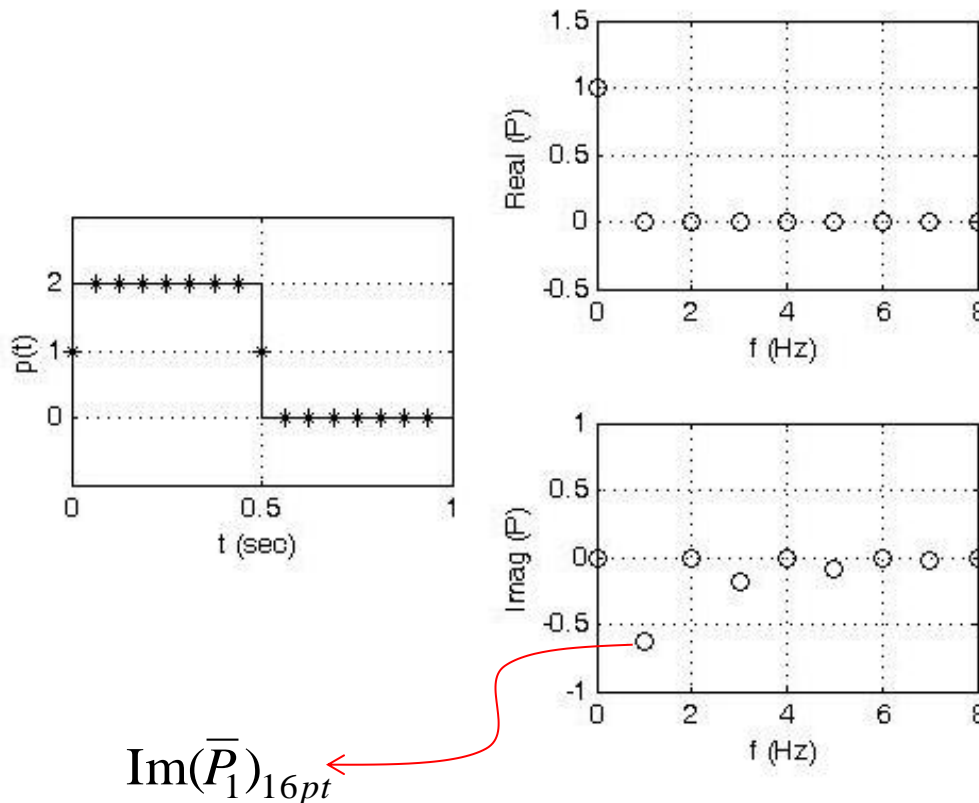
- ❑ Various algorithms have been developed to make calculations efficient/fast.
- ❑ Fast Fourier Transform (FFT) is one of the most efficient methods that compute the Discrete Fourier Transform. FFT is not a new type of transform but rather, an efficient numerical algorithm for evaluating DFT.

$$A_m = \sum_{n=0}^{N-1} B_n W_N^{mn} \quad (m = 0, 1, \dots, N-1) \quad W_N = e^{-i(2\pi/N)}$$

- ❑ Total N^2 complex products are required to evaluate A_m ($m=0, 1, \dots, N$). But due to the cyclic nature of powers of W_N , the total computational time can be drastically reduced. That is, the total number of complex products for the FFT is $(N/2)\log_2 N$.
- ❑ For example, if $N=512$, then FFT operation is less than 1% of the original operations of DFT.
- ❑ **In conclusion, for FFT algorithm, choose a power of 2 (2, 4, 8, ..., 1024, 2048, 4096...) for the number of frequency lines.**

Example 5

- Using FFT command in MATLAB to compute a 16-point Fourier transform to verify the results shown in Example 3. Note p_0 is 1. Compare $|\bar{P}_1|_{16pt}$ $|\bar{P}_1|_{32pt}$ $|\bar{P}_1|_{64pt}$ with the result from Example 3.



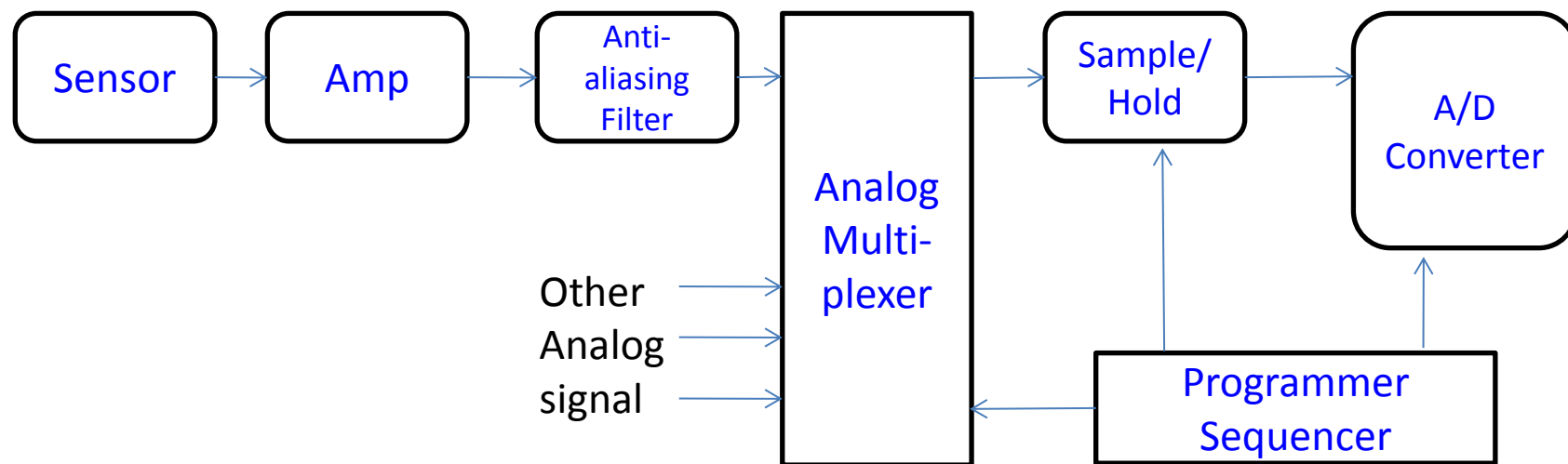
$ \bar{P}_1 $	Value
Example 3	0.636619772
$ \bar{P}_1 _{16pt}$	0.628417436
$ \bar{P}_1 _{32pt}$	0.634573149
$ \bar{P}_1 _{64pt}$	0.636108363

As the number of sampling points increase, the value gets closer to analytical solution from Example 3

6

Digital Signal Processing

❑ System Configuration for A/D Signal Conversion

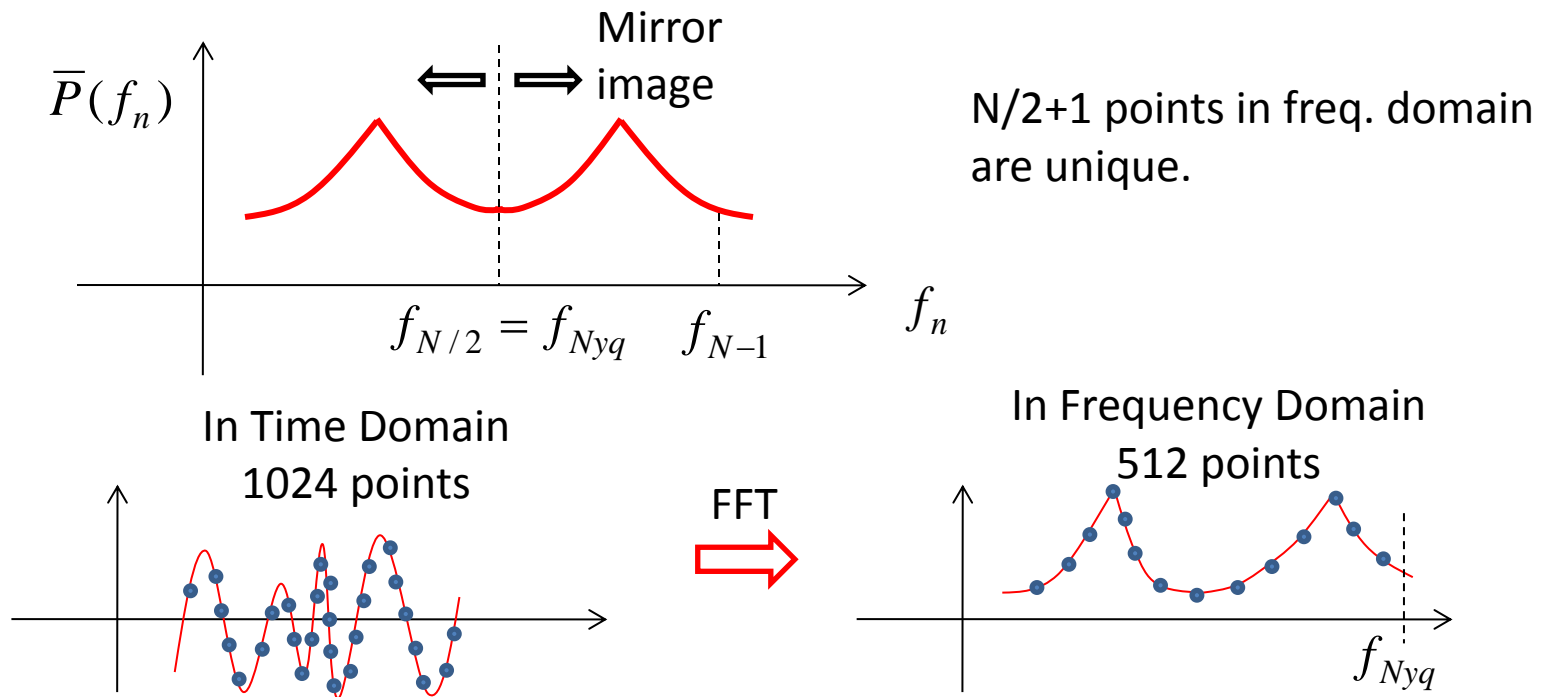


- ❑ Sensors output small magnitudes of voltage signals, for example, a few mV. Therefore, the amplifier is used to amplify the signals. But it could amplify the noise too.
- ❑ It is important to convert physical signals to digital signals without loss of information.

- Shannon's sampling theory tells that sampling frequency (f_s) should be at least twice larger than max frequency (f_0) of our interests.

$$f_s \geq 2f_0$$

- Because frequency above $f_{N/2}$ (**called Nyquist frequency: half of the sampling frequency**) cannot be observed in the data, those values are only unique to $\bar{P}(f_{N/2})$. Above $f_{N/2}$, the results are mirror image.



- ❑ If the time interval Δt is constant and the total sampling period T_1 increases, more points will be generated in frequency domain (i.e. higher resolution in frequency domain). But the frequency bandwidth will be the same.
- ❑ If the time interval Δt decreases and the total sampling period T_1 is constant, more points will be generated in frequency domain. However, since bandwidth increases, we will have the same frequency resolution.

Example 6

- When $\Delta f = 5\text{Hz}$ and 1024 samples are taken,

$$T = \frac{1}{\Delta f} = \frac{1}{5} = 0.2 \text{ sec}$$

$$f_s = \frac{N}{T} = N\Delta f = 5120 \text{ Hz}$$

$$f_{Nyq} = \frac{f_s}{2} = 2560 \text{ Hz}$$

- When $f_c = 50 \text{ kHz}$ and a total of 4096 samples are taken,

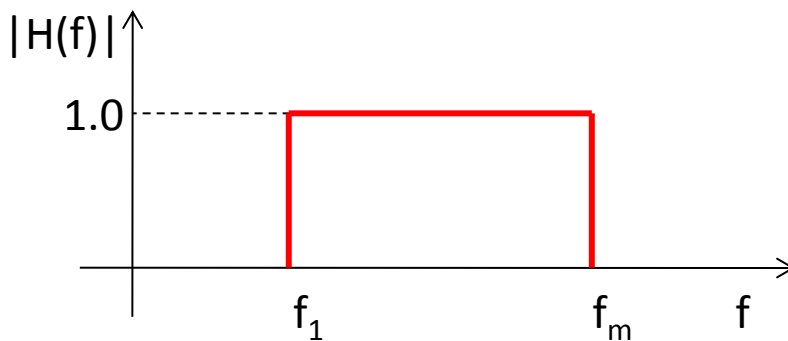
$$f_s = 2f_c = 100 \text{ kHz}$$

$$\Delta f = \frac{1}{N\Delta t} = \frac{f_s}{N} = 24.4 \text{ Hz}$$

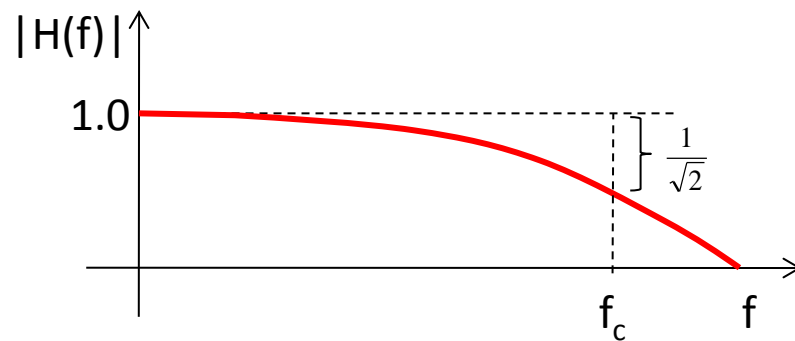
$$T = \frac{1}{\Delta f} = 0.04 \text{ sec}$$

Anti-aliasing Filter

- During sampling, unwanted signals could be included due to aliasing effect.
- Since frequencies greater than f_0 occurs aliasing, **anti-aliasing filter** (a kind of low pass filter) should be used to remove the higher frequencies.
- The sampling frequency should be twice the max frequency in theory. However, considering that damping characteristics of anti-aliasing filter, it should be 2.56 or 4 times of the max frequency.



Anti-aliasing filter



But actual anti-aliasing filter looks like this

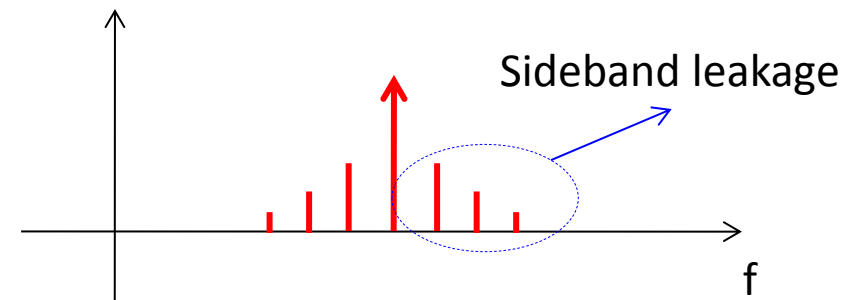
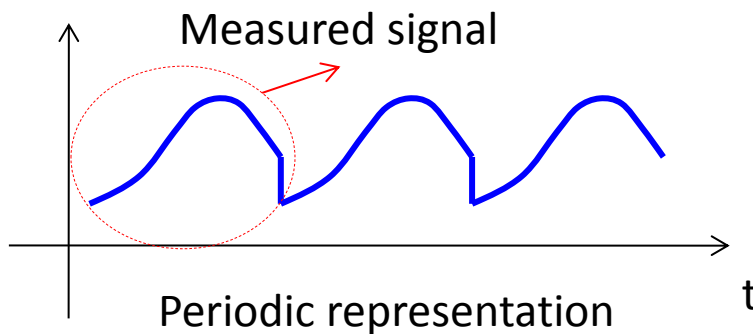
A/D Conversion (Quatization)

- ❑ Depending on the number of bits in A/D converter, the resolution is determined.
- ❑ The resolution (R) is determined as

$$R = A / 2^n$$

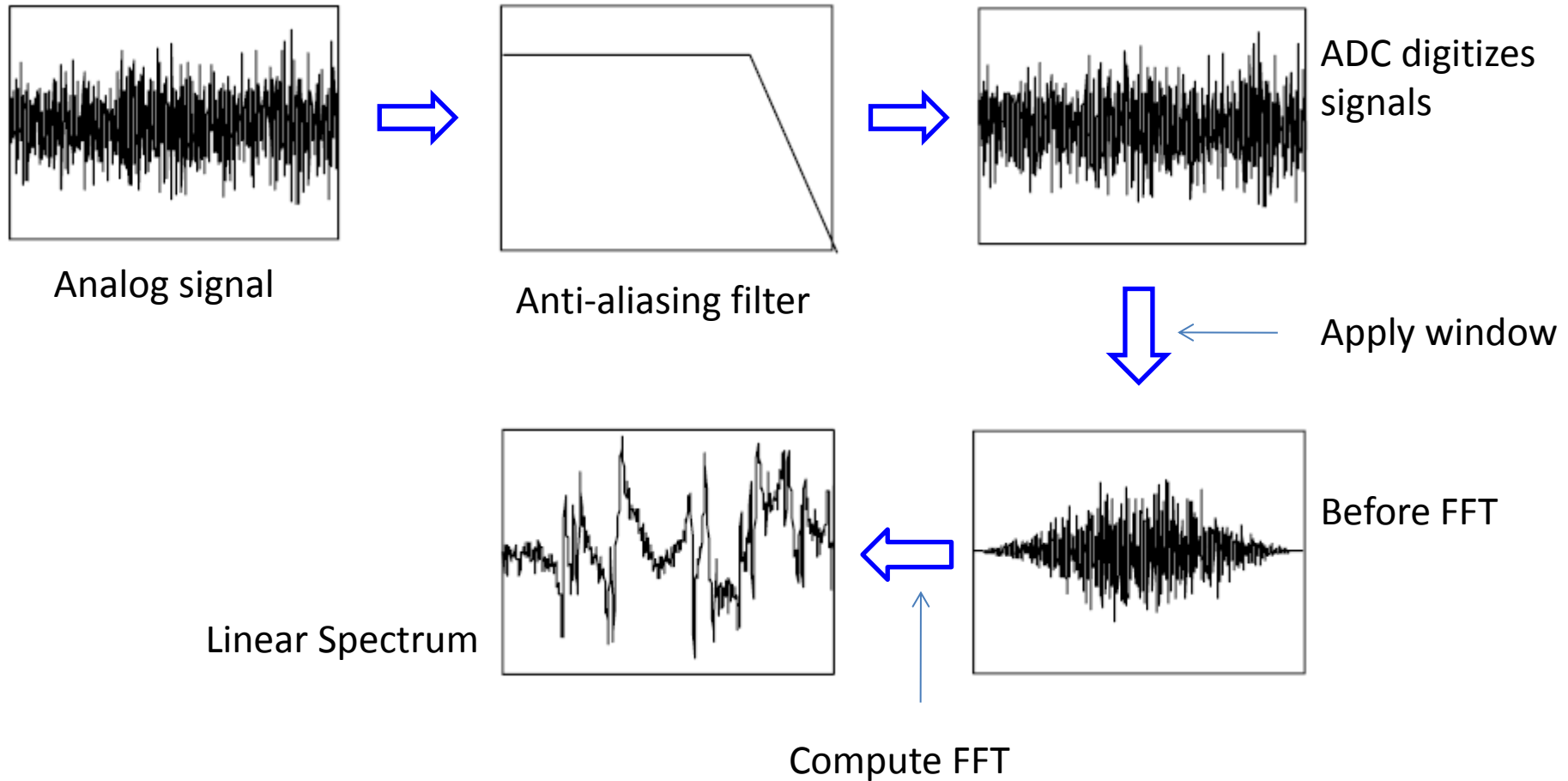
- ❑ Where A is the peak-to-peak value of voltage output and n is the number of bits of the selected A/D converter.
-

- ❑ Notes on FFT
 - ❑ Increasing N will increase the resolution of the FFT for constant Δt
 - ❑ Decreasing Δt will increase the max frequency obtained (bandwidth)
 - ❑ Typically averaging is necessary to get good results
- ❑ DFT assumes that the sampled signal is infinitely long and periodic. Notice that there are discontinuity in the periodic version of this signal. This discontinuity introduce additional frequency components into the frequency domain.



- ❑ Thus, windowing is used to minimize these effects
- ❑ Time domain segment is multiplied by a “window” before taking FFT.
- ❑ Window function used to continuous signals
 - ❑ Square window
 - ❑ Hanning window
 - ❑ Hamming window
 - ❑ Kaiser-Bessel window
 - ❑ Flat-top window
 - ❑ User-defined window

Digital Signal Processing Flow Diagram



7

Analyses of Digital Signals From NI manuals

- ❑ For 1D signals, FFT.VI computes the Discrete Fourier Transform (DFT) of the input signals with a FFT algorithm.
- ❑ Each frequency component is a dot product of the time-domain signal with the complex exponential at that frequency.

$$x(k) = \sum_{n=1}^N x(n) e^{-j \left(\frac{2\pi n k}{N} \right)} = \sum_{n=1}^N x(n) \left[\cos \left(\frac{2\pi n k}{N} \right) - j \sin \left(\frac{2\pi n k}{N} \right) \right]$$

$$F_{\max} = f_s / 2$$

$$\Delta f = 1/T = f_s / N$$

Δf is the frequency resolution and T is the acquisition time,
 f_s is the sampling frequency and N is the block size of the FFT

- ❑ It returns the double-sided power spectrum of X.
- ❑ NI Power Spectrum.VI uses the FFT & DFT routine to compute the Power Spectrum.

$$\text{Power Spectrum } S_{xx} = \frac{|F\{X\}|^2}{n^2} = X^*(f)X(f) = |X(f)|^2$$

S_{xx} represents the output sequence Power Spectrum,

n is the number of samples in the input sequence X

$$n = 2^m \quad (m = 1, 2, 3, \dots, 23)$$

Unit of powerspectrum $((\text{input signal unit})_{rms})^2$

$S(f)$ represents the output sequence powerspectrum

Δf is the frequency resolution and T is the acquisition time

f_s is the sampling frequency and N is the block size of the FFT

- ❑ It computes the single-sided, scaled, auto power spectrum of time-domain signals.
- ❑ It computes the power spectrum using the following equation.

$$Power\ Spectrum = \frac{FFT^*(Signal) \times FFT(Signal)}{n^2}$$

n is the number of points in the signal and

$*$ denotes the complex conjugate

- ❑ It converts the power spectrum into a single-sided power spectrum.

- ❑ Two steps to compute the single-sided and scaled amplitude spectrum.
- ❑ Using the following equation, it computes two-sided amplitude spectrum.

$$A(i) = \frac{X(i)}{N}, \quad i = 1, 2, 3, \dots, N$$

A is the two - sided amplitude spectrum

X is the discrete Fourier transform of signal

N is the number of points in signal

- ❑ Based on following equation, it converts the two-sided amplitude spectrum to the single-sided amplitude spectrum.

$$B(j) = \begin{cases} A(1) & i = 1 \\ \sqrt{2}A(i) & i = 2, 3, \dots, \left\lfloor \frac{N}{2} - 1 \right\rfloor \end{cases}$$

B is the single - sided amplitude spectrum, $\lfloor \cdot \rfloor$ is the floor operation

Amp Spectrum Mag = $|B|$

Amp Spectrum Phase = $Phase(B)$

- ❑ FFT of a real signal is a complex number, having real & imaginary parts.
- ❑ Power in each frequency component represented by the FFT can be acquired by squaring the magnitude of that frequency component
- ❑ The power in the k^{th} frequency component defined as

$$\text{Power} = |X(k)|^2$$

$|X(k)|$: Magnitude of the frequency component

- ❑ Power spectrum returns an array that contains the two-sided power spectrum of a time-domain signal.
- ❑ Power Spectrum shows the power in each of frequency components.
- ❑ The equation below compute the two-sided power spectrum from FFT.

$$\text{Power Spectrum } S_{AA}(f) = \frac{\text{FFT}(A) \times \text{FFT}^*(A)}{N^2}$$

A : Time Domain Signal

$\text{FFT}^*(A)$: Complex Conjugate of $\text{FFT}(A)$

Δf is the frequency resolution and T is the acquisition time

f_s is the sampling frequency and N is the block size of FFT

- ❑ PSD shows the strength of variations(energy) as a function of frequency.
- ❑ PSD shows at which frequencies variations are strong and at which frequencies variations are weak.
- ❑ The unit of PSD is energy per frequency (width).
- ❑ Computation of PSD is done directly by the method called FFT or computing autocorrelation function and then transforming it.

$$PSD \ S(f) = \int_{-\infty}^{\infty} R(\tau) e^{-2\pi i f \tau} d\tau, \text{ Autocorrelation function } R(\tau) = \frac{E[(X_t - \mu)(X_{t+\tau} - \mu)]}{\sigma^2}$$

E is expected value, μ is mean, σ is variance and τ is the lag

- ❑ Steps of computing FFT PSD
 - ❑ Compute the FFT of time signals
 - ❑ Form the PS or PSD of time signals
 - ❑ Average with next computations

FFT Spectrum(Mag-Phase).VI

- ❑ The FFT Spectrum (Mag-Phase) can compute Magnitude and Phase parts.
- ❑ FFT Spectrum (Mag-Phase) can compute averaged spectrum of time signals.
 - ❑ It computes the FFT of time signals.
 - ❑ It averages the current FFT spectrum of time signals with the FFT spectra computed by the VI since the last time the averaging process was reset.
 - ❑ It returns the Mag-Phase parts of the averaged spectrum.

FFT Spectrum(Real-Im).VI

- ❑ The FFT Spectrum (Real-Im) can compute real and imaginary parts.
- ❑ FFT Spectrum (Real-Im) can compute averaged spectrum of time signals.
 - ❑ It computes the FFT of time signals.
 - ❑ It averages the current FFT spectrum of time signal with the FFT spectra computed by the VI since the last time the averaging process was reset.
 - ❑ It returns the real and imaginary parts of the averaged spectrum.

8

Hands-on Experiment Project

Problem Statement (1)

- ❑ **Conduct vibration test** using a beam structure and measure dynamic response.



- ❑ **Conduct FFT-based signal processing** of the acquired data using the provided NI-LabVIEW VI program.

Guides for Reports

- ❑ Write a full report using the instructions provided in class. Organize your report into sections (e.g. Introduction, Procedures, Results, Discussion, Summary, References). Write concisely and clearly.
 - ❑ Include the following: (1) A schematic diagram and description of the test equipment. (2) Plots of the time-domain vibration response data measured from the experiment. (3) Plots of various signal processing measures computed by the NI-LabVIEW VI program.
-

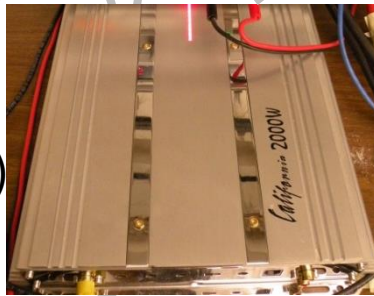
9

Test Setup and Equipment

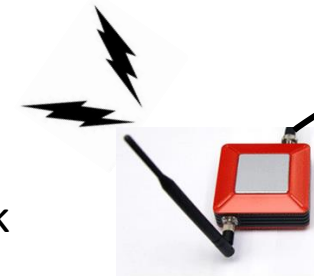
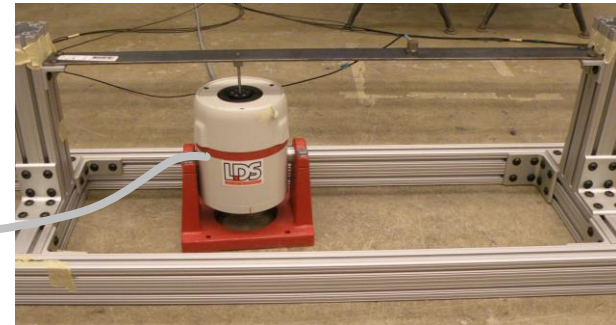
Test Setup and Procedures (1)



Power Supply(MPJA 14604PS)



Amplifier(AP2000)



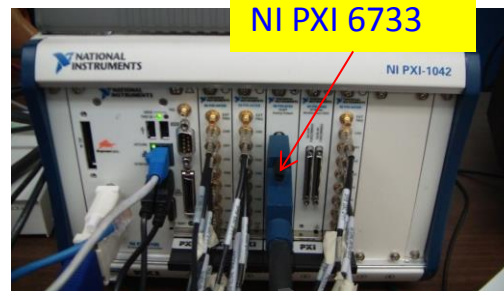
AnyLoggerS-V/ICP



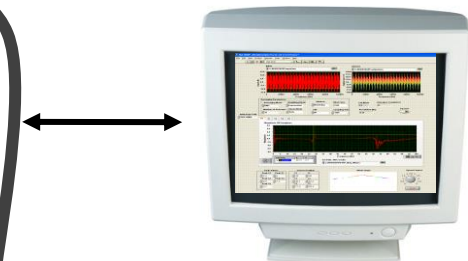
Connector Block
(SCB68)



MSP-100



NI-PXI 8105 in NI-PXI 1042

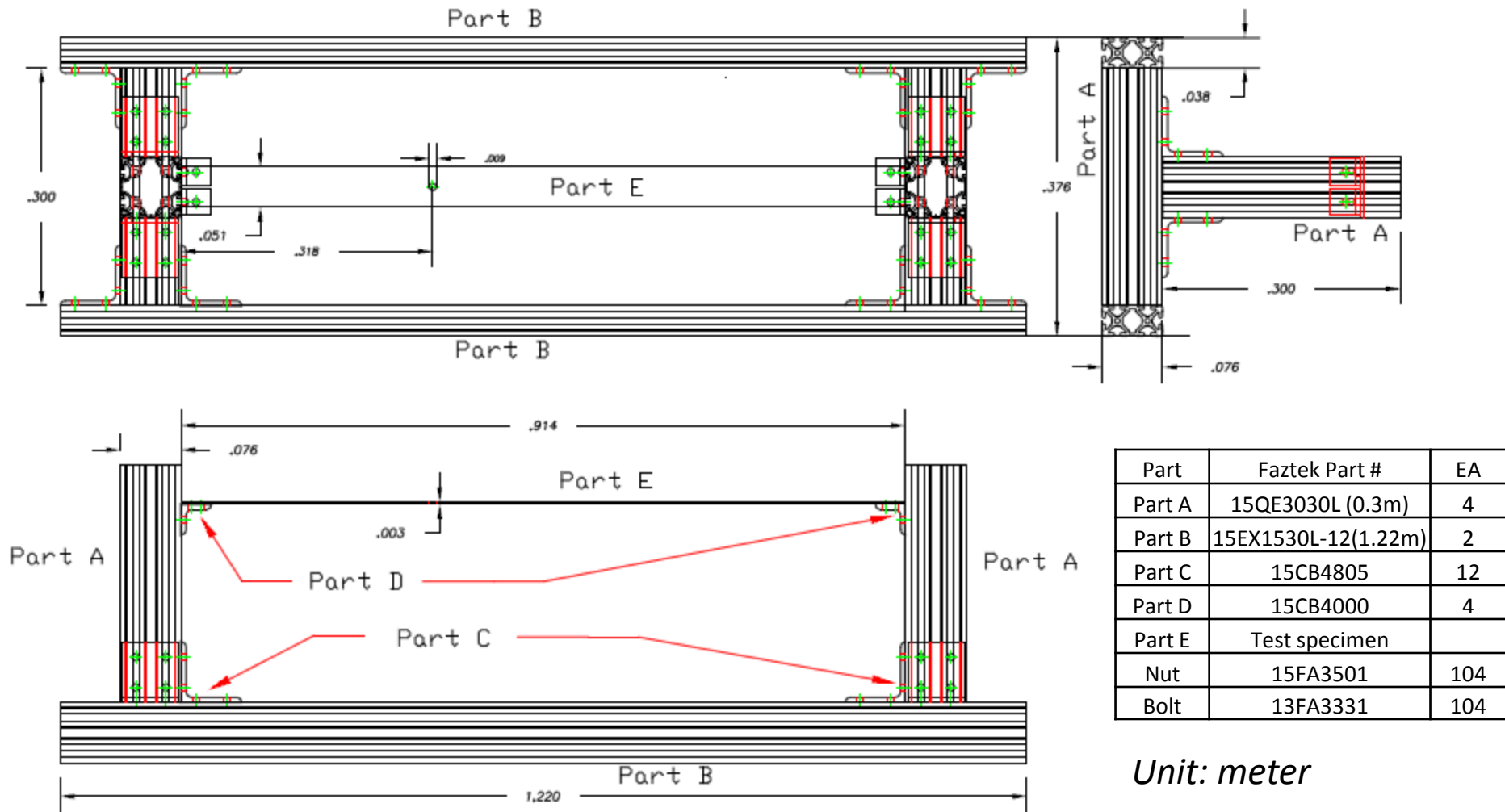


NI- LabVIEW VI Program

Test Setup and Procedures (2)

- ❑ Step1 Turn on power supply.
- ❑ Step2 Generate excitation signals using NI-LabVIEW and send them through analog output board (NI-PXI 6733).
- ❑ Step3 Under the vibration excitation, proceed the test.

Test Structure- Drawing



Part	Faztek Part #	EA
Part A	15QE3030L (0.3m)	4
Part B	15EX1530L-12(1.22m)	2
Part C	15CB4805	12
Part D	15CB4000	4
Part E	Test specimen	
Nut	15FA3501	104
Bolt	13FA3331	104

Test Equipment

- ❑ Agilent 33250A (For function generator based test)
 - ❑ Amplifier (California AP2000 2000W)
 - ❑ Power Supply (MPJA 14604PS)
 - ❑ NI-PXI 8105 Controller
 - ❑ NI-PXI 6733 (For NI analog out signal generator based test)
 - ❑ 68-Pin Connector Block (SCB68)
 - ❑ ICP type Dytran triaxial accelerometer (3093B1)
 - ❑ One-channel AnyLogger (Korea Maintenance Co., LTD): AnyLoggerS-V/ICP.
 - ❑ NI-LabVIEW 8.6
-

Specification



- ☐ 320 watts RMS x 2 at 4 ohms
- ☐ 480 watts RMS x 2 at 2 ohms
- ☐ 960 watts RMS x 1 bridged output at 4 ohms
- ☐ 4-ohm stable in bridged mode
- ☐ Stereo or bridged mono output
- ☐ Tri-way capable (Tri-Way Crossover required)
- ☐ Dual power supply for stability at high volumes
- ☐ Fuse rating: 25A x 4
- ☐ Requires 4-gauge power and ground leads and a 100-amp fuse
- ☐ Wiring and hardware not included with amplifier
- ☐ Variable low-pass filters (50-250 Hz, 12 dB/octave)
- ☐ Variable bass boost (0-12 dB) at 45 Hz
- ☐ Variable subsonic filter (20-50 Hz)
- ☐ Preamp-level inputs (speaker-level to preamp-level adapter included)
- ☐ Preamp outputs
- ☐ Wired bass level remote control
- ☐ 24-1/4"W x 2"H x 10-1/4"D

Specification



- ☐ Input voltage : 110-127 AC
- ☐ Output voltage : 0-30 DC
- ☐ Current : 0-10 A
- ☐ Voltage regulation
 - ☐ CV $\square 1 \times 10^{-4} + 3\text{mV}$
 - ☐ CC $\square 2 \times 10^{-3} + 6\text{mA}$
- ☐ Load regulation
 - ☐ CV $\square 5 \times 10^{-4} + 3\text{mV}$
 - ☐ CC $\square 5 \times 10^{-4} + 6\text{mA}$
- ☐ Ripple & noise
 - ☐ CV $< 1.5\text{mVrms}$
 - ☐ CC $< 10\text{mArms}$
- ☐ Protection : current limiting
- ☐ Voltage indication accuracy : $1\% + 1\text{d}$
- ☐ Current indication accuracy : $1\% + 1\text{d}$
- ☐ Ambient temperature : $0 \sim 40^\circ\text{C}$
- ☐ Humidity: $< 90\%$

Specification

- ❑ Intel Core Duo Processor T2500(2.0 GHz dual core)
- ❑ 512 MB (1 x 512 MB DIMM) dual channel 667 MHz DDR2 RAM standard, 4 GB (2 x 2 GB DIMMs) maximum
- ❑ Integrated I/O
 - ❑ 10/100/1000BASE-TX Ethernet
 - ❑ 4 Hi-Speed USB ports
 - ❑ ExpressCard/34 slot
 - ❑ DVI-I video connector
 - ❑ GPIB (IEEE 488) controller
 - ❑ RS232 serial port
 - ❑ IEEE 1284 ECP/EPP parallel port
 - ❑ Integrated hard drive
- ❑ Internal PXI trigger bus routing
- ❑ Watchdog timer Software
- ❑ Hard drive-based recovery image PXI System



Specification

- ❑ 8 high-speed digital I/O lines; two 24-bit counters; digital triggering
- ❑ Onboard or external update clock PXI trigger bus for synchronization with DAQ motion, and vision products
- ❑ NI DAQmx driver with configuration utility to simplify configuration and measurement
- ❑ Superior integration: LabVIEW, LabVIEW Real-Time, LabWindows™/CVI, and Measurement Studio for VB
- ❑ 1MS/s, 16-Bit, 8 Channels



68-Pin Connector Block (SCB68)

Specification



- ☐ Number of channels : 8 differential, 16 single-ended
- ☐ Accuracy : $\pm 1.0^{\circ}\text{C}$ over a 0° to 110°C range
- ☐ Output : $10\text{ mV}/^{\circ}\text{C}$
- ☐ I/O connectors One 68-pin male SCSI connector
- ☐ Temperature : 0° to 70°C
- ☐ Relative humidity : 5% to 90% non-condensing
- ☐ Temperature : -55° to 125°C
- ☐ Relative humidity : 5% to 90% non-condensing

Dytran Accelerometer (3093B1)



Model 3093B1 Dytran Triaxial Accelerometer

Specification	Value	Unit
Weight	13.5	Grams
Size(Height x Width x Depth)	0.54 x 0.59 x 0.59	Inch
Sensitivity	100	mV/G
Ranges	+/-50	G
Frequency Response	2 to 5000	Hz
Equivalent Electrical Noise	0.007	G, RMS
Linearity	1	% F.S.
Temp. Range	-60 ~ +250	°F
Supply Current Range each axis	2 to 20	mA
Supply Voltage Range each axis	+18 to +30	VDC
Output impedance	100	OHMS



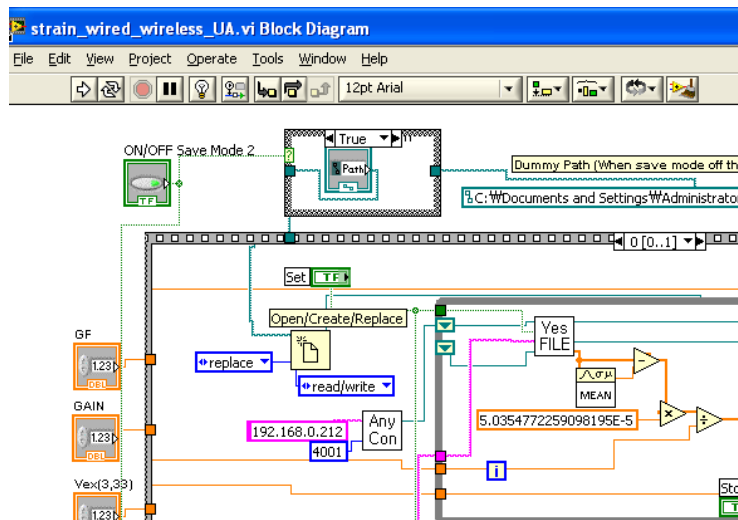
AnyloggerS-V/ICP for acceleration transmitter
AnyLoggerS-B for strain transmitter

Specification

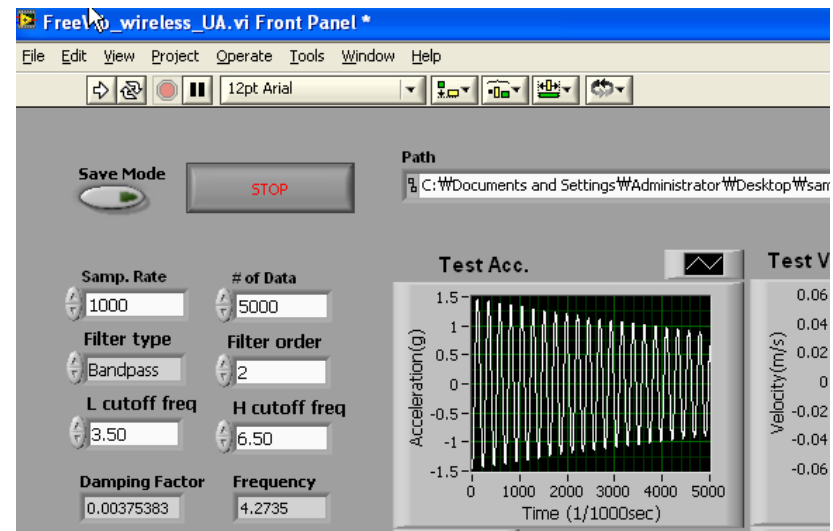
Contents	AnyloggerS-V/ICP	AnyLoggerS-B
Support Num. of Channel	1	1
Input Voltage Range	-5 ~ 5V	0 ~ 3V
Gain	1,2,5,10,20,50,100	50,100,250,500,1000,2500,5000
Programmable	10~1000Hz (10,20,50,100,200,500,1000)	10 ~ 1000Hz
Lower Pass Filter		10,20,50,100,200,500,1000
Prog. Offset	0 ~ 5V(12Bit)	0 ~ 3.3V(12Bit)
Max Sampling Rate	1000Hz	1000Hz
Exciting Voltage	24V(Only ICP Type)	3.3V±0.5%
Connector	BNC Connector	4Pin Circular Connector
SIM Usage	No	Yes
Power Consumption (w/o sensor)	150mA	100mA(w/o SIM)
Internal Battery	Li-ion Rechargeable 1500mAh x 2EA(Serial)	Li-ion Rechargeable 1500mAh x 2EA(Serial)
Ext. Power Requirement	5V	5V
Operation Temperature	-10 ~ 80℃	-10 ~ 80℃
Sync. Accuracy	< 10ms	< 10ms
Dimension	800 x 973 x 353	800 x 883 x 353
Weight	210g	210g
ADC Resolution	Differential 16Bit	Differential 16Bit
Measurement Accuracy	F.S. 0.1%	F.S. 0.1%
Sensor Connectivity	Voltage or ICP source	Bridge type sensor
Signal Ripple	Depends on Gain	Depends on Gain
Communication	Bluetooth v1.2 class1 18dBm(w/o ant.)	Bluetooth v1.2 class1 18dBm(w/o ant.)
Radio Frequency Range	2.402 ~ 2.480GHz	2.402 ~ 2.480GHz
Transmission Method	FHSS(freq. Hopping Spread Spectrum)	FHSS
Modulation Method	GFSK(Gaussian-filtered Freq. Shift Keying)	GFSK
Approvals	MIC, FCC, CE	MIC, FCC, CE

❑ LabVIEW (Laboratory Virtual Instrument Engineering Workbench) is a graphical programming language that uses icons instead of lines of text to create applications.

❑ In contrast to text-based programming languages, where instructions determine the order of program execution, LabVIEW uses dataflow programming, where the flow of data through the nodes on the block diagram determines the execution order of the VIs and functions. VIs, or virtual instrument, are LabVIEW programs that imitate physical instruments.



VI Block Diagram



Front Panel

10

Users Manual of NI-LabVIEW VI Program

Users Manual of LabVIEW VI Prog. (1)

- 1) **Set Parameter** – to “Monitoring Start” and click “Set”
- 2) **Set Path** – Create an empty file in “Path” where the original raw Acc. data will be saved
- 3) **Select Mode “RAW DATA SAVE MODE”** – Save the original raw data to the file
- 4) **Sampling Rate** – Default 1KHz
- 5) **Run**- Run the front panel while the structure is vibrating
- 6) **Stop** – After more than specified times (# of samples*/ Samp. Rate), click “Stop”. Users can control the monitoring time.

***Note:** Users should get enough # of samples for subsequent signal analyses.

FFTPSandmore22.vi Front Panel

File Edit View Project Operate Tools Window Help

12pt Arial

5 6 1 2 3 4

Parameter
Monitoring Start

Path
C:\test1\testdata.lvm

Select Mode
RAW DATA SAVE MODE

Samp. Rate
1000

Select Analysis Type
FFT

Input Parameter

dt: 0.001, # of sample: 1000, dB On (F): OFF
 window: Hanning, FFT size: -1

Unit and Operation Table

Analysis Name	Input Parameter	Unit of Graph1	Unit of Graph2
FFT	# of sample, FFT size	X:Hz Y:g pk	X:Hz Y:rad
Power Spectrum	# of sample	X:Hz Y:(g rms)^2	none
Auto Power Spectrum	# of sample, dt	X:Hz Y:(g rms)^2	none
Amp. & Phase Spectrum	# of sample, dt	X:Hz Y:g rms	X:Hz Y:rad
FFT Power Spectrum	# of sample, dB On, window	X:Hz Y:(g rms)^2	none
FFT Spectral Density	# of sample, dB On, window	X:Hz Y:(g rms)^2/Hz	none
FFT Spectral(Mag-Phase)	# of sample, window	X:Hz Y:g rms	X:Hz Y:rad
FFT Spectral(Real-Imag)	# of sample, window	X:Hz Y:none	X:Hz Y:none

Time Domain Signal

Acceleration(g)

Time(sec)

Acceleration

Graph1

Amplitude

Frequency (Hz)

Graph2

Phase Angle(rad)

Frequency (Hz)

Graph1

Graph2

Users Manual of LabVIEW VI Prog. (2)

After getting raw data, users can run eight kinds of signal processing analyses using this front panel.

At first "FFT analysis" is carried out .

7) Select Mode – Select "ANALYSIS MODE"

8) Select Analysis Type – Select FFT analysis

9) Set path – Create empty files in the folder where the acceleration data, graph1 data and graph2 data will be saved. Users can use file extensions such as *.txt, *.dat or *.lvm

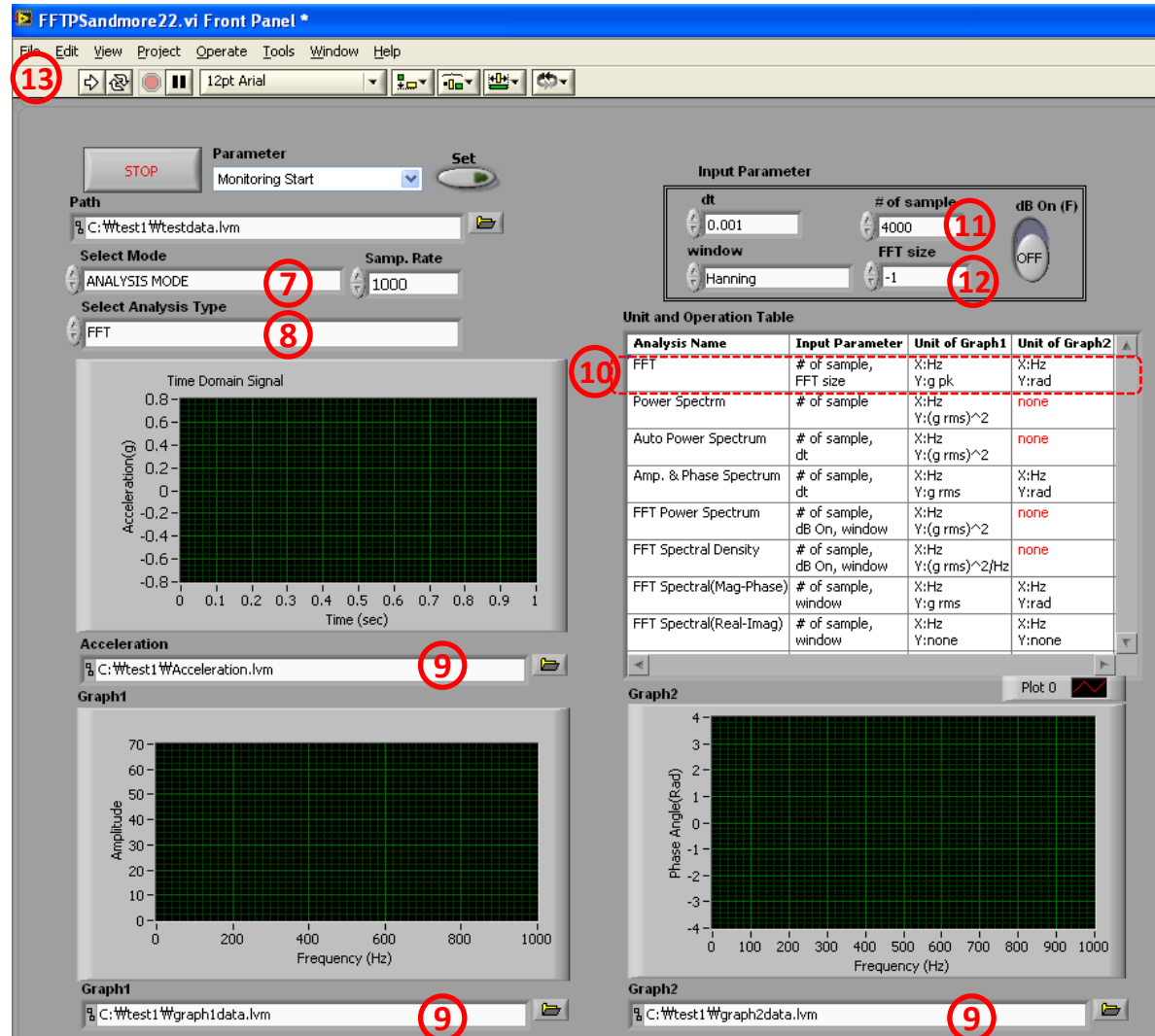
10) Unit and Operation table- Check the table to find out the parameter needed for analysis and the unit of the results.

11) # of sample –Enter the number of samples for analysis

Note - Number of samples should be less than the number of original raw data which was acquired by the user at the "RAW DATA SAVE MODE"

12) Set FFT size* default is -1 which means it uses all the samples in $x(t)$. See explanations from NI.

13) Run

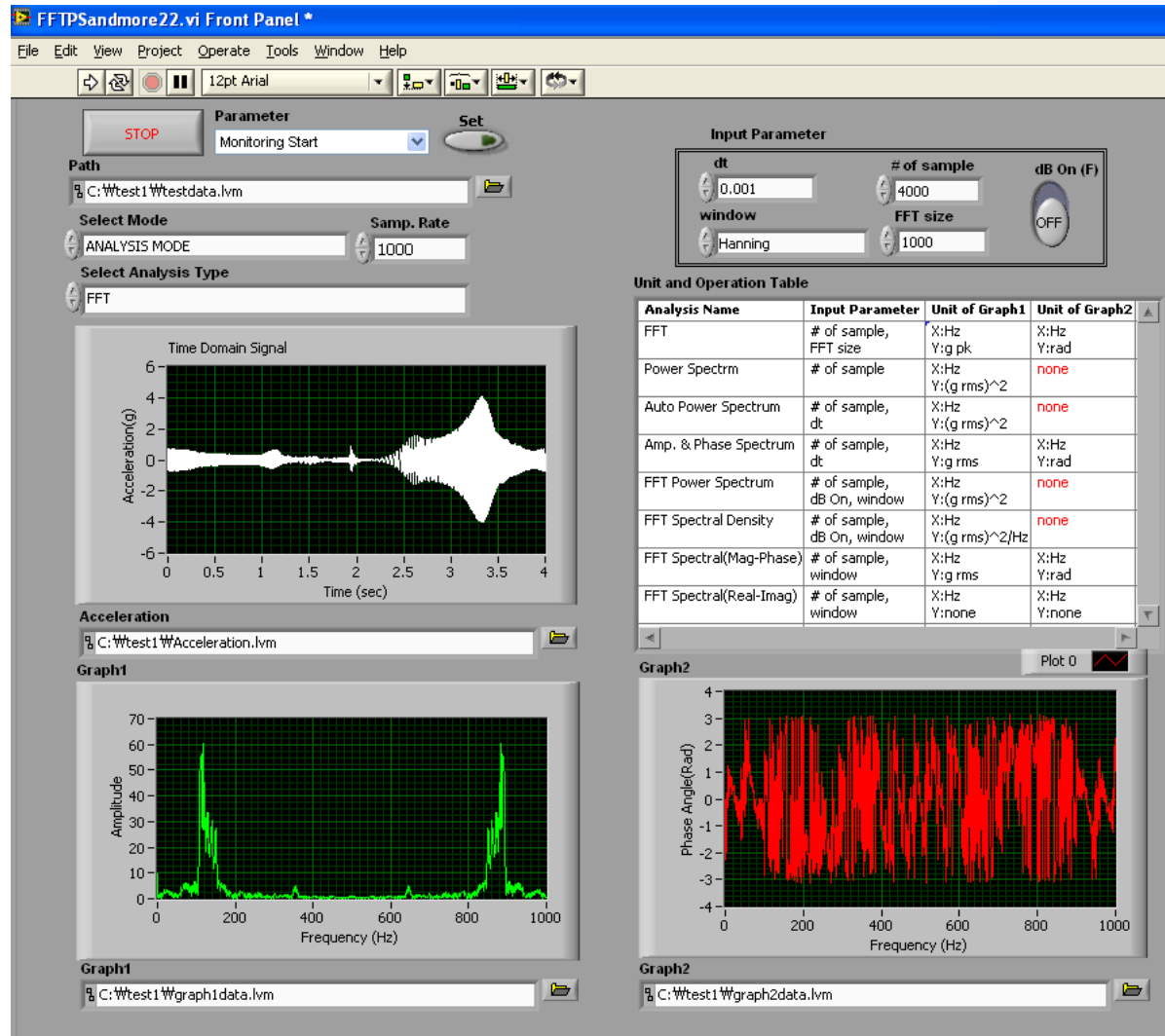


Unit and Operation Table

Analysis Name	Input Parameter	Unit of Graph1	Unit of Graph2
FFT	# of sample, FFT size	X:Hz Y:g pk	X:Hz Y:rad
Power Spectrm	# of sample	X:Hz Y:(g rms)^2	none
Auto Power Spectrum	# of sample, dt	X:Hz Y:(g rms)^2	none
Amp. & Phase Spectrum	# of sample, dt	X:Hz Y:g rms	X:Hz Y:rad
FFT Power Spectrum	# of sample, dB On, window	X:Hz Y:(g rms)^2	none
FFT Spectral Density	# of sample, dB On, window	X:Hz Y:(g rms)^2/Hz	none
FFT Spectral(Mag-Phase)	# of sample, window	X:Hz Y:g rms	X:Hz Y:rad
FFT Spectral(Real-Imag)	# of sample, window	X:Hz Y:none	X:Hz Y:none

* **FFT size** is the length of the FFT you want to perform. If **FFT size** is greater than the number of elements in **X**, this VI adds zeros to the end of **X** to match the size of **FFT size**. If **FFT size** is less than the number of elements in **X**, this VI uses only the first n elements in **X** to perform the FFT, where n is **FFT size**. If **FFT size** is less than or equal to 0, this VI uses the length of **X** as the **FFT size**.

Results of FFT Analysis



Users Manual of LabVIEW VI Prog. (4)

As an additional analysis example, "FFT Spectrum(Real-Imag) analysis" is carried out. User can run the other analysis continuously based on the same original raw data

14) Select Analysis Type

Select FFT Spectrum(Real-Imag)

15) Unit and Operation table- Check the table to find out the parameter required for analysis and the unit of the results

16) # of sample –Enter the number of samples for analysis

Note - Number of samples should be less than the number of original raw data which was acquired by the user at the "RAW DATA SAVE MODE"

17) Window–Select the type of window

18) Set path – Create different name of empty files in the folder where the graph1 data and graph2 data will be saved. Users can use those file extensions such as *.txt, *.dat or *.lvm

19) Run

FFTSandmore22.vi Front Panel *

File Edit View Project Operate Tools Window Help

12pt Arial

STOP Parameter Monitoring Start Set

Path C:\test1\Wtestdata.lvm

Select Mode ANALYSIS MODE Samp. Rate 1000

Select Analysis Type FFT 14

Input Parameter

dt 0.001 # of sample 4000 16 dB On (F) OFF

window Hanning 17 FFT size 1000

Unit and Operation Table

Analysis Name	Input Parameter	Unit of Graph1	Unit of Graph2
FFT	# of sample, FFT size	X:Hz Y:g pk	X:Hz Y:rad
Power Spectrm	# of sample	X:Hz Y:(g rms)^2	none
Auto Power Spectrum	# of sample, dt	X:Hz Y:(g rms)^2	none
Amp. & Phase Spectrum	# of sample, dt	X:Hz Y:g rms	X:Hz Y:rad
FFT Power Spectrum	# of sample, dB On, window	X:Hz Y:(g rms)^2	none
FFT Spectral Density	# of sample, dB On, window	X:Hz Y:(g rms)^2/Hz	none
FFT Spectral(Mag-Phase)	# of sample, window	X:Hz Y:g rms	X:Hz Y:rad
FFT Spectral(Real-Imag)	# of sample, window	X:Hz Y:none	X:Hz Y:none

Time Domain Signal

Acceleration(g)

Time (sec)

Acceleration C:\test1\WAcceleration.lvm

Graph1

Amplitude

Frequency (Hz)

Graph2

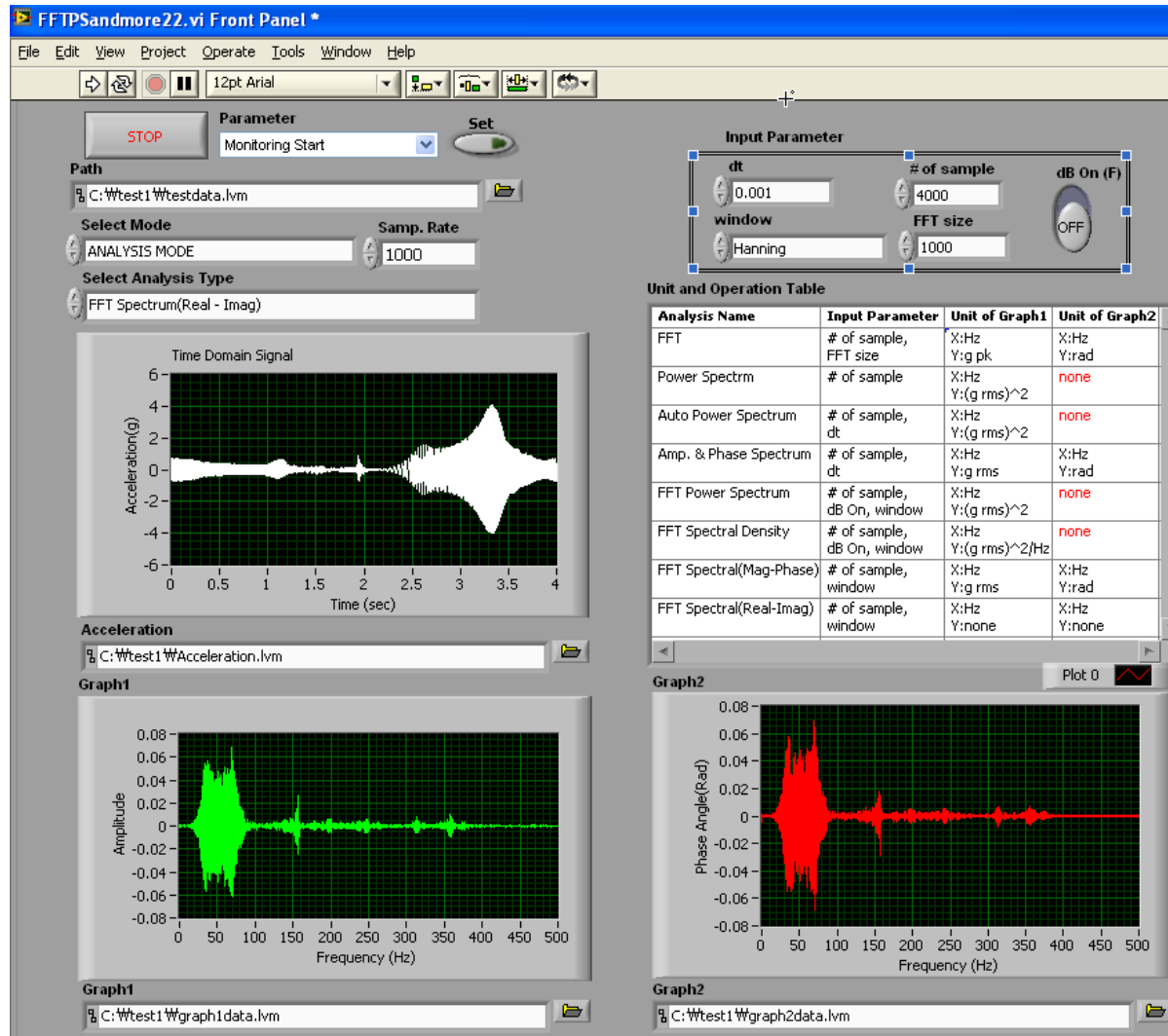
Phase Angle(Rad)

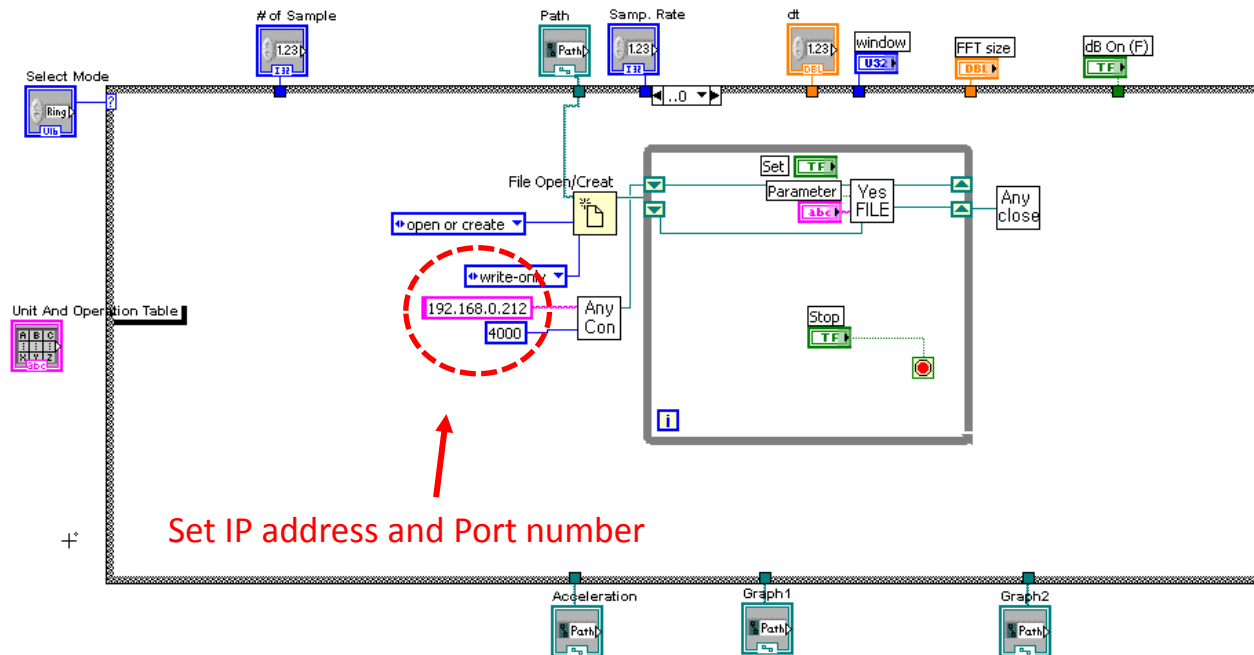
Frequency (Hz)

Graph1 C:\test1\Wgraph1data.lvm 18

Graph2 C:\test1\Wgraph2data.lvm 18

Results of FFT Spectrum(Real-Imag) Analysis



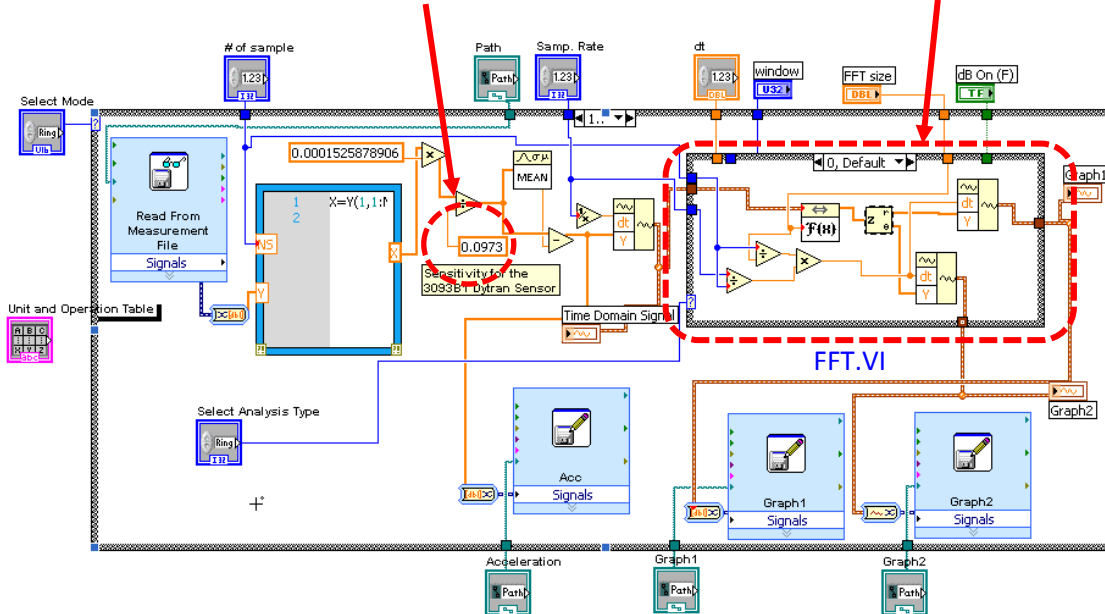


First Stack of Block Diagram of VI Program

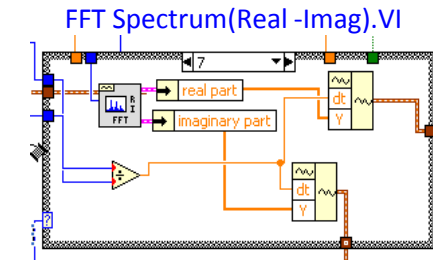
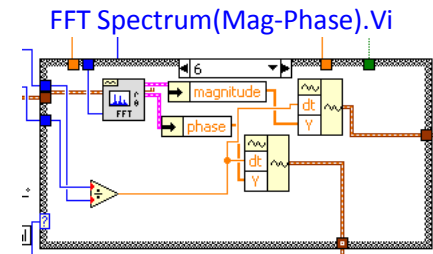
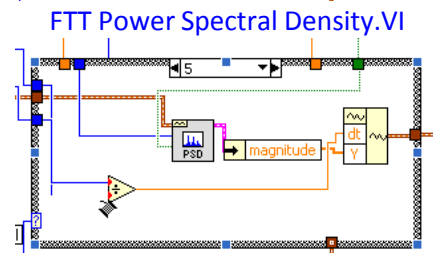
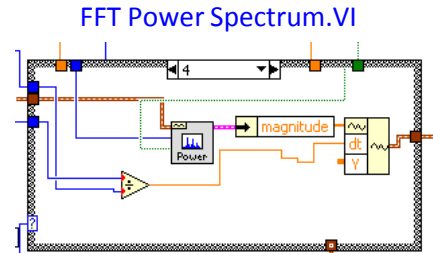
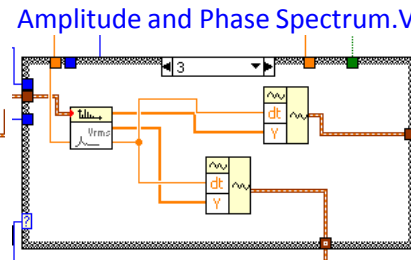
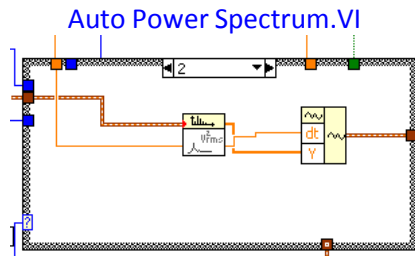
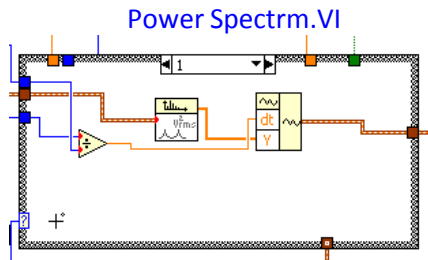
- ❑ Stack Sequence and Case Structure were used
- ❑ The first sequence, program saves the raw data at the given path
- ❑ Users should define correct **IP address** and **port number** based on the equipments following the ANYLOGGER[®] manual.

Sensitivity of accelerometer (V/g)

Eight Stack Sequences



Second Stack of Block Diagram of VI Program



- ❑ In the second frame, users need to set the sensitivity of sensor.
- ❑ All eight stack sequences were implemented corresponding to the analysis types.

- For better understanding of digital signal processing, a NI-LabVIEW VI program is provided to process simulated sinusoidal signals.

Users Manual of LabVIEW VI Prog. (9)

- 1) **Sine Signal Generator** – set frequency, amplitude and sampling rate to generate sine waves
- 2) **Select Analysis Type** – Users can select one of the eight analysis types. Set "FFT Spectral(Mag-Phase)
- 3) **Unit and Operation table** –Check the table to find out the parameter needed for analysis (# of samples and window) and the unit of the output graphs
- 4) **Set # of samples** – Based on the number of samples, the program generate sine wave and do analysis(for example 1000)
- 5) **Set window** Set the window as Hanning
- 6) **Set path** – Create empty files in the folder where the acceleration data, graph1 data and graph2 data will be saved and set path. User can use those file extensions such as *.txt, *.dat or *.lvm
- 7) **Run**

FFT_PSD_sim.vi Front Panel

File Edit View Project Operate Tools Window Help

12pt Arial

Sine Signal Generator 1

Frequency: 30 Amplitude: 10 Samp. Rate: 1000

Select Analysis Type 2

FFT

Time Domain Signal

Acceleration(g)

Time (sec)

Acceleration

C:\Wfftpsd\Wacc.lvm 6

Graph1

Amplitude

Frequency (Hz)

Graph1

C:\Wfftpsd\Wgraph1.lvm 6

Input Parameter

dt: 0.001 # of sample: 1000 dB On (F): OFF

window: Hanning 5 FFT size: 1000

Unit and Operation Table

Analysis Name	Input Parameter	Unit of Graph1	Unit of Graph2
FFT	# of sample, FFT size	X:Hz Y:g pk	X:Hz Y:rad
Power Spectrm	# of sample	X:Hz Y:(g rms)^2	none
Auto Power Spectrum	# of sample, dt	X:Hz Y:(g rms)^2	none
Amp. & Phase Spectrum	# of sample, dt	X:Hz Y:g rms	X:Hz Y:rad
FFT Power Spectrum	# of sample, dB On, window	X:Hz Y:(g rms)^2	none
FFT Spectral Density	# of sample, dB On, window	X:Hz Y:(g rms)^2/Hz	none
FFT Spectral(Mag-Phase)	# of sample, window	X:Hz Y:g rms	X:Hz Y:rad
FFT Spectral(Real-Imag)	# of sample, window	X:Hz Y:none	X:Hz Y:none

3

Graph2

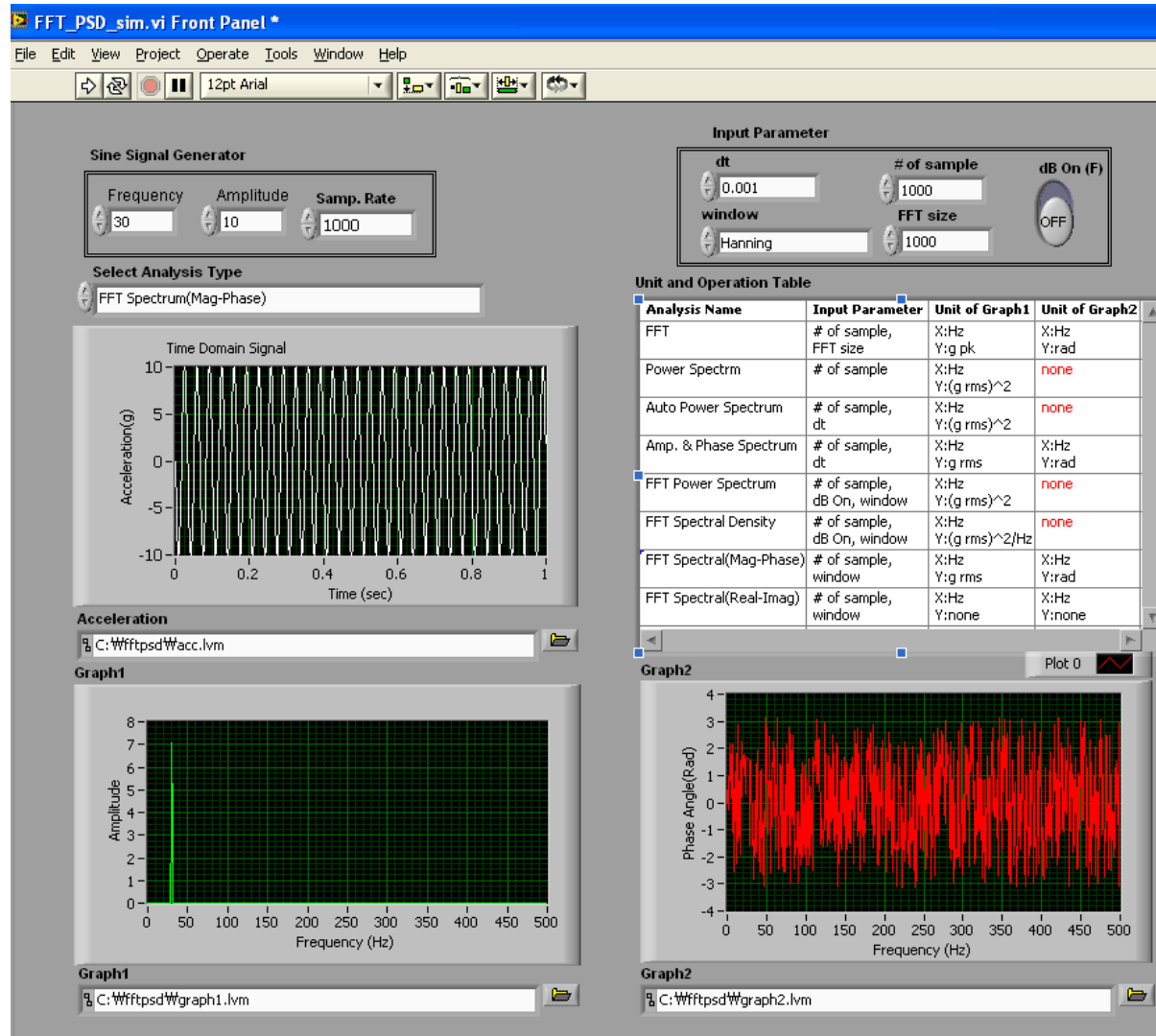
Phase Angle(Rad)

Frequency (Hz)

Graph2

C:\Wfftpsd\Wgraph2.lvm 6

Results of FFT Spectrum (Mag-Phase)



Users Manual of LabVIEW VI Prog. (11)

As an additional analysis example, "Amplitude and Phase Spectrum analysis" is carried out. Users can run the other analysis continuously based on the sine wave which is generated by the user

8) Select Analysis Type

Select Amp. and Phase Spectrum

9) Unit and Operation table- Check the table to find out the parameter needed for analysis and the unit of the results

10) # of sample –Based on the number of samples, the program generate sine wave and do analysis(for example 5000)

Note – Users can change other parameter but there will be no effect for analysis such as FFT size or window

11) dt – Input the dt (1/sampling rate)

12) Set path – Create different name of empty files in the folder where the graph1 data and graph2 data will be saved and set path. User can use those file extensions such as *.txt, *.dat or *.lvm

13) Run

FFT_PSD_sim.vi Front Panel *

File Edit View Project Operate Tools Window Help

13

Sine Signal Generator

Frequency: 30 Amplitude: 10 Samp. Rate: 1000

Select Analysis Type

8 FFT Spectrum(Mag-Phase)

Time Domain Signal

Acceleration(g)

Time (sec)

Acceleration

C:\Wfftspd\Wacc.lvm

Graph1

Amplitude

Frequency (Hz)

12

Input Parameter

dt: 0.001 11

of sample: 1000 10

dB On (F): OFF

window: Hanning

FFT size: 1000

Unit and Operation Table

Analysis Name	Input Parameter	Unit of Graph1	Unit of Graph2
FFT	# of sample, FFT size	X:Hz Y:g pk	X:Hz Y:rad
Power Spectrm	# of sample	X:Hz Y:(g rms)^2	none
Auto Power Spectrum	# of sample, dt	X:Hz Y:(g rms)^2	none
9! Amp. & Phase Spectrum	# of sample, dt	X:Hz Y:g rms	X:Hz Y:rad
FFT Power Spectrum	# of sample, dB On, window	X:Hz Y:(g rms)^2	none
FFT Spectral Density	# of sample, dB On, window	X:Hz Y:(g rms)^2/Hz	none
FFT Spectral(Mag-Phase)	# of sample, window	X:Hz Y:g rms	X:Hz Y:rad
FFT Spectral(Real-Imag)	# of sample, window	X:Hz Y:none	X:Hz Y:none

Graph2

Phase Angle(Rad)

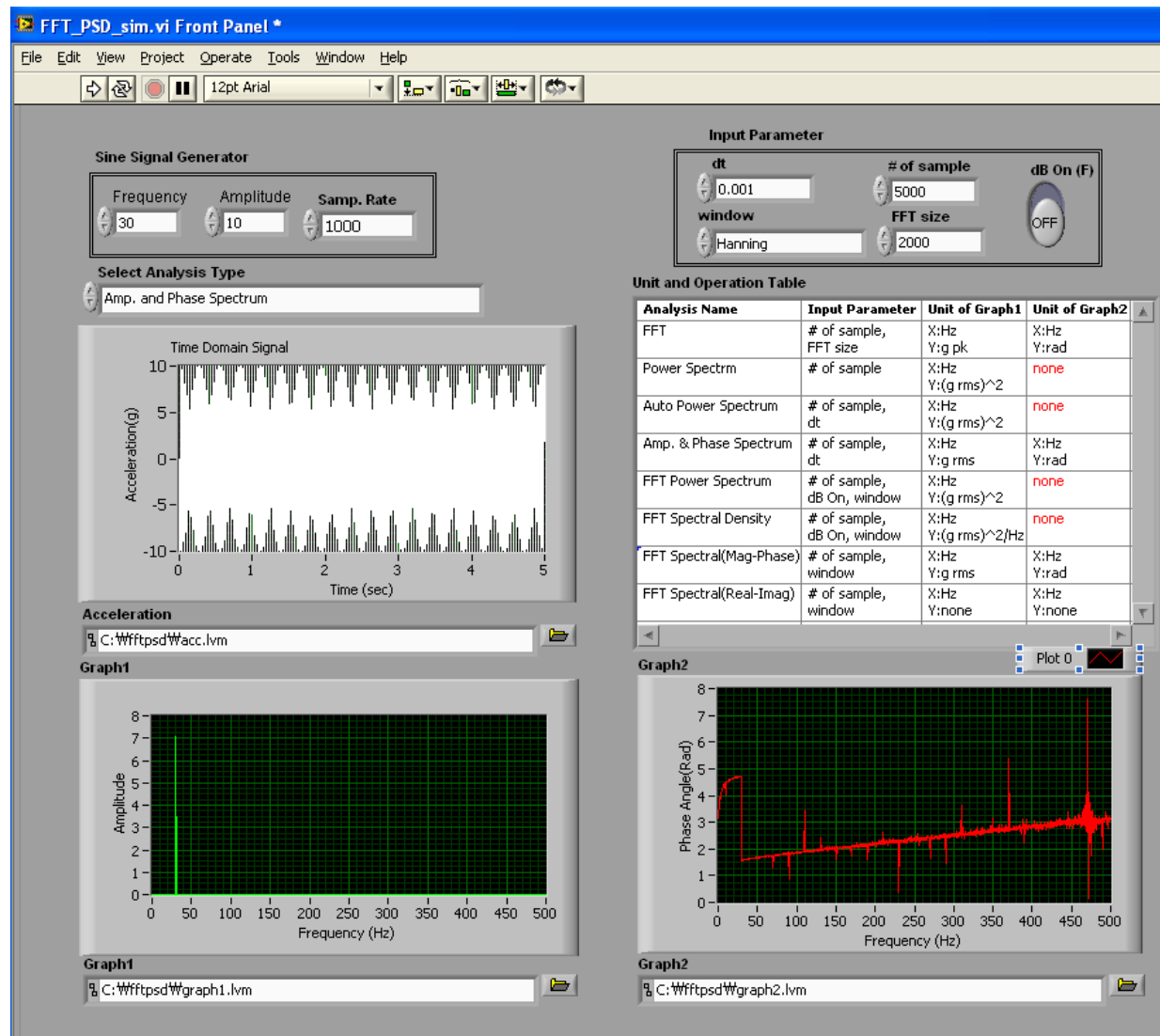
Frequency (Hz)

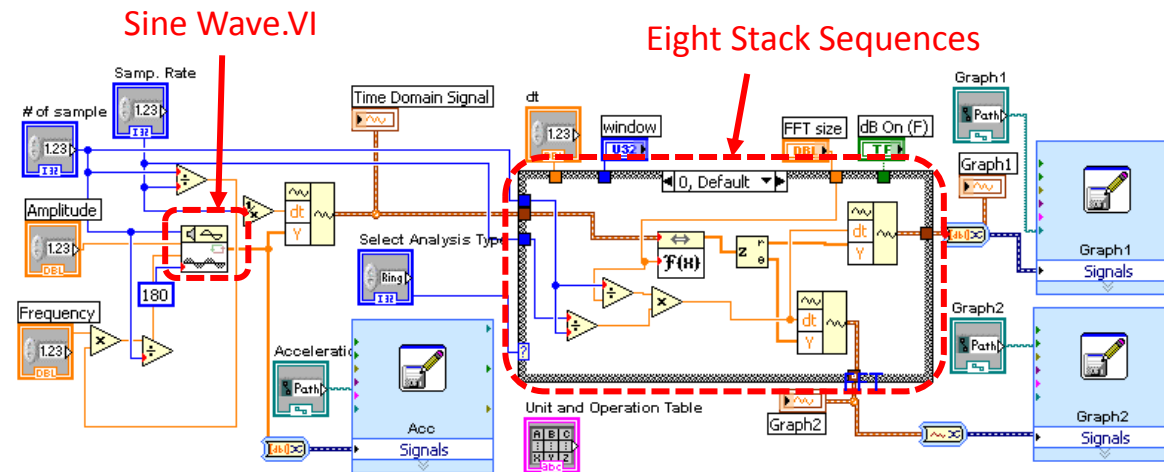
12

C:\Wfftspd\Wgraph1.lvm

C:\Wfftspd\Wgraph2.lvm

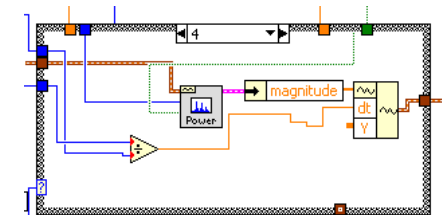
Results of Amp. and Phase Spectrum



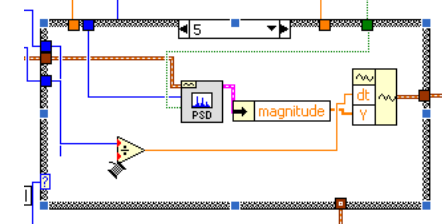


Block Diagram of VI Program

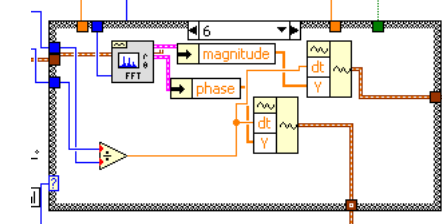
FFT Power Spectrum.VI



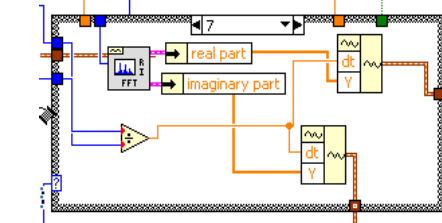
FFT Power Spectral Density.VI



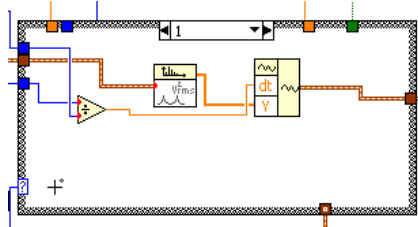
FFT Spectrum(Mag-Phase).Vi



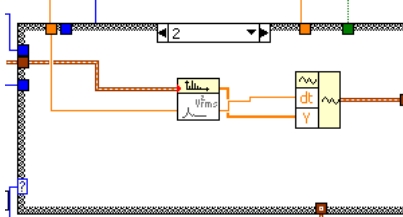
FFT Spectrum(Real -Imag).Vi



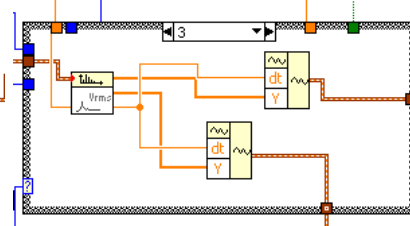
Power Spectrm.VI



Auto Power Spectrum.VI



Amplitude and Phase Spectrum.VI



- ❑ Sine Wave.VI was used for generate sine wave
- ❑ All eight stack sequences were implemented corresponding to the analysis types.

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References

- Roy R. Craig, Jr. and Andrew J. Kurdila, "Fundamentals of Structural Dynamics", 2nd Ed. Wiley
 - Anil K. Chopra, " Dynamics of Structures-Theory and Applications to Earthquake Engineering", 2nd Ed. Prentice Hall
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