



# Lecture 6: Experimental Modal Testing of MDOF System

1. Learning Objectives
  2. Vibration of Undamped 2DOF System
  3. Introduction of Mode Superposition with 2 DOF System
  4. Vibration Properties of MDOF System: Mode, and Frequency
  5. Frequency-Response Analysis of MDOF System
  6. Experimental Modal Testing
  7. Hands-on Experiment Project
  8. Test Setup and Equipment
  9. Users Manual of NI-LabVIEW VI Program
  10. Reference
-

1

# Learning Objectives

# Learning Objectives

---



- ❑ **Objective 1:** Determine modal properties of the MDOF system.
  - ❑ **Objective 2:** Determine dynamic response of MDOF system using the mode superposition.
  - ❑ **Objective 3:** Experimentally determine modal properties from frequency response function (FRF).
-



# 2

## Vibration of Undamped 2 DOF System

---

# Undamped 2DOF System



- For an undamped 2 DOF system, the equation of motion takes a form

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

- Assuming the harmonic response,

$$u_1(t) = U_1 \cos(\omega t - \alpha) \quad \ddot{u}_1(t) = -U_1 \omega^2 \cos(\omega t - \alpha)$$

$$u_2(t) = U_2 \cos(\omega t - \alpha) \quad \ddot{u}_2(t) = -U_2 \omega^2 \cos(\omega t - \alpha)$$

$$\left( \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \right) \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{----- (1)}$$

- To have a non-trivial solution

$$\det \left( \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \right) = 0 \quad \longrightarrow \quad \text{Characteristic Equation}$$

# Undamped 2DOF System



- Solve for the roots

$$\omega_1^2 \text{ and } \omega_2^2 \Rightarrow \text{Eigen values}$$

$$f_1 = \frac{\omega_1}{2\pi} \text{ and } f_2 = \frac{\omega_2}{2\pi}$$

- Substituting  $\omega = \omega_1$  to either of the equations (1) and obtaining the ratio

$$\beta_1 = \left( \frac{U_2}{U_1} \right)^{(1)}$$

- Substituting  $\omega = \omega_2$  to either of the equations (1) and obtaining the ratio

$$\beta_2 = \left( \frac{U_2}{U_1} \right)^{(2)}$$

$$\vec{\phi}_1 = \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}_1 = A_1 \begin{Bmatrix} 1 \\ \beta_1 \end{Bmatrix}$$

- Then the mode shapes are found as

$$\vec{\phi}_2 = \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}_2 = A_2 \begin{Bmatrix} 1 \\ \beta_2 \end{Bmatrix}$$

# Undamped 2DOF System

---



- Then, the general solution for the response is a linear combination of these two and can be written as

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} = A_1 \begin{Bmatrix} 1 \\ \beta_1 \end{Bmatrix} \cos(\omega_1 t - \alpha_1) + A_2 \begin{Bmatrix} 1 \\ \beta_2 \end{Bmatrix} \cos(\omega_2 t - \alpha_2)$$

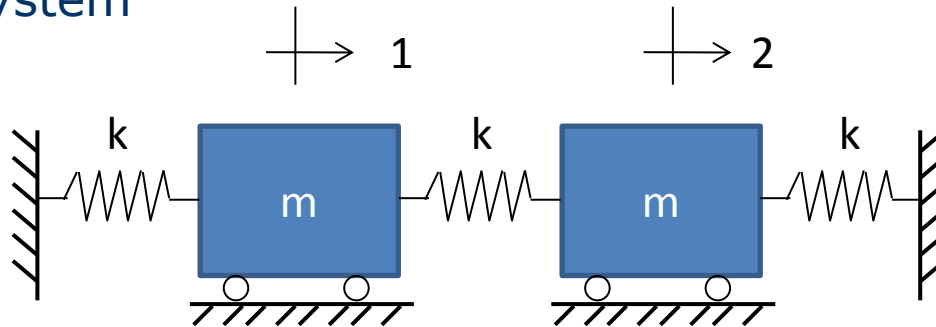
- $A_1$ ,  $A_2$ ,  $\alpha_1$  and  $\alpha_2$  are determined from initial conditions.



# Example I (1)



- Compute the natural frequencies and mode shapes of the 2-DOF system



$$m = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \quad k = \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}$$

$$u_1(t) = U_1 \cos(\omega_i t - \alpha)$$

$$u_2(t) = U_2 \cos(\omega_i t - \alpha)$$

$$\det \left( \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} - \omega_i^2 \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \right) = 0 \quad \left| \begin{array}{cc} 2k - \omega_i^2 m & -k \\ -k & 2k - \omega_i^2 m \end{array} \right| = 0$$

$$(2k - \omega_i^2 m - k)(2k - \omega_i^2 m + k) = 0$$

$$\omega_1 = \sqrt{\frac{k}{m}} \quad \text{and} \quad \omega_2 = \sqrt{\frac{3k}{m}} \quad \omega_1 < \omega_2$$

## Example I (2)



- Substituting  $\omega_1$  into  $([k] - \omega_i^2[m])\{u\} = 0$

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}_1 = 0 \quad \longrightarrow \quad U_1 = U_2$$

$$\vec{\phi}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \quad \text{symmetric mode}$$

- Substituting  $\omega_2$  into  $([k] - \omega_i^2[m])\{u\} = 0$

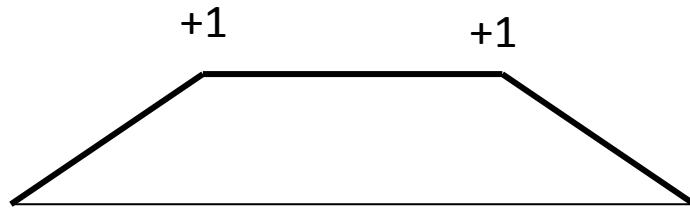
$$\begin{bmatrix} -k & -k \\ -k & -k \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}_2 = 0 \quad \longrightarrow \quad U_1 = -U_2$$

$$\vec{\phi}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \quad \text{unsymmetric mode}$$

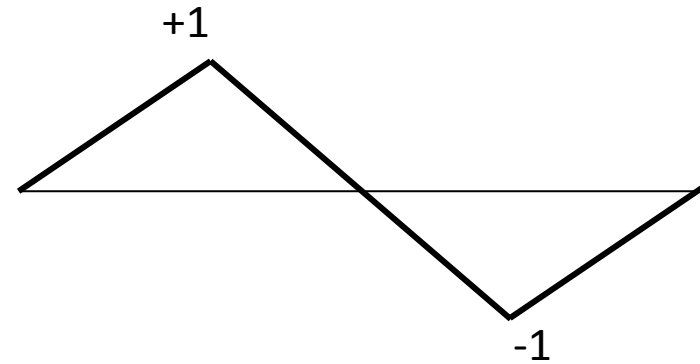
# Example I (3)



- Visualizing the mode shape



Mode I

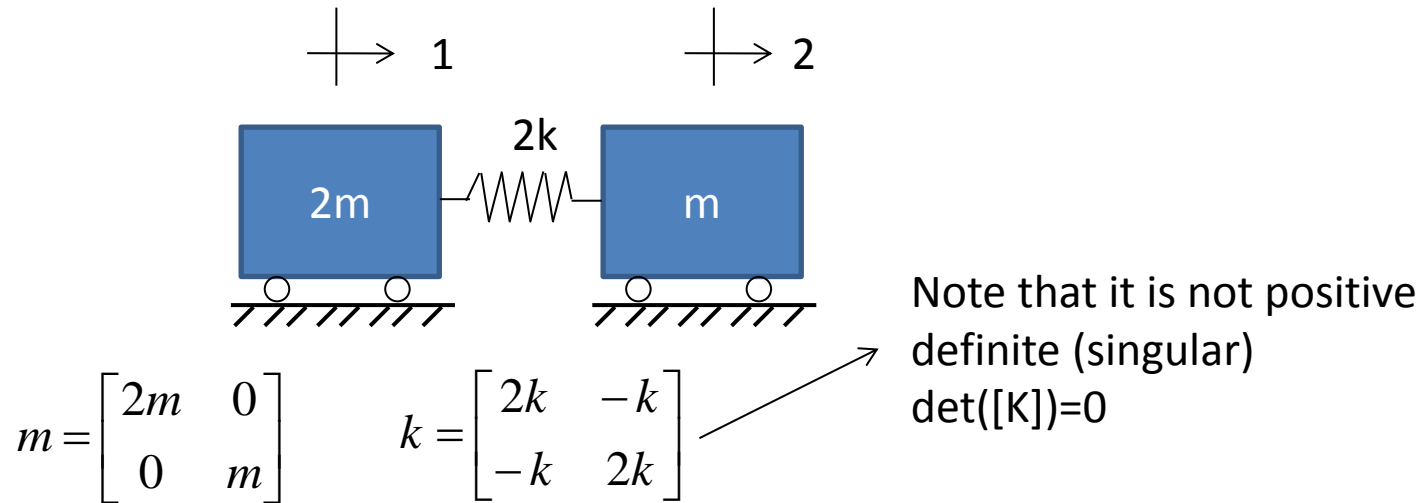


Mode II

## Example II (1)



- Consider a system that is under free vibration with a rigid-body mode



- Assuming the harmonic response

$$u_1(t) = U_1 \cos(\omega t - \alpha)$$

$$u_2(t) = U_2 \cos(\omega t - \alpha)$$

- The characteristic equation is

$$\det \left( \begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix} - \omega_i^2 \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \right) = 0 \quad \begin{vmatrix} 2k - 2\omega_i^2 m & -2k \\ -2k & 2k - \omega_i^2 m \end{vmatrix} = 0$$

## Example II (2)



$$(2k - 2m\omega_i^2)(2k - m\omega_i^2) - 4k^2 = 0$$

$$2m^2\omega_i^4 - 6mk\omega_i^2 = 0$$

$$m\omega_i^2(m\omega_i^2 - 3k) = 0 \quad \omega_1 = 0 \quad \text{and} \quad \omega_2 = \sqrt{\frac{3k}{m}}$$

- Substituting  $\omega_1$  into  $([k] - \omega_i^2[m])\{U\} = 0$

$$\begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}_1 = 0 \quad \vec{\phi}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}_1$$

- Substituting  $\omega_2$  into  $([k] - \omega_i^2[m])\{U\} = 0$

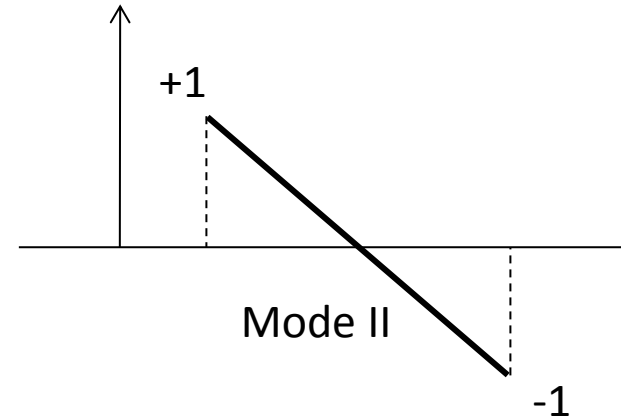
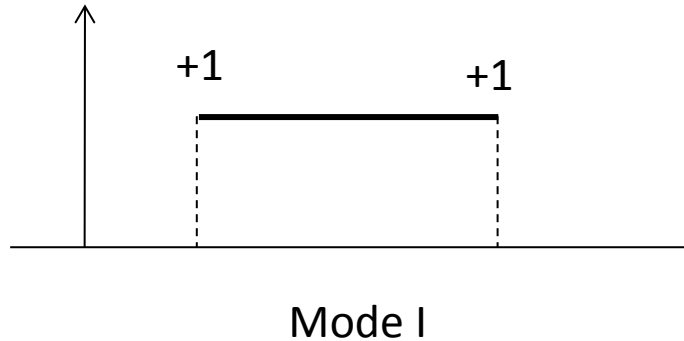
$$\left( \begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix} - \begin{bmatrix} 6k & 0 \\ 0 & 3k \end{bmatrix} \right) \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}_2 = 0$$

$$\begin{bmatrix} -4k & -2k \\ -2k & -k \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}_2 = 0 \quad \vec{\phi}_2 = \begin{Bmatrix} 1 \\ -2 \end{Bmatrix}_2$$

## Example II (3)



- Visualizing the mode shapes,



- Zero eigenvalue means rigid body motion and the stiffness matrix is singular.  $|k|=0$



# 3

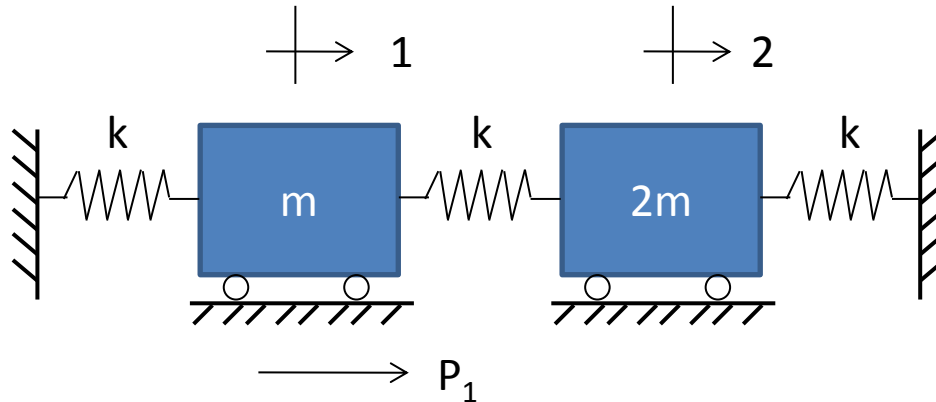
## Introduction to Mode Superposition with 2 DOF System

---

# Introduction to Mode Superposition



- Let's consider two masses connected with springs. The equation of motion is as follows



$$m \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + k \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ 0 \end{Bmatrix} \cos \Omega t$$

$$\omega_1^2 = \frac{k}{m} \quad \vec{\phi}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \quad \text{and} \quad \omega_2^2 = \frac{5}{2} \left( \frac{k}{m} \right) \quad \vec{\phi}_2 = \begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix}$$

$$\Phi = [\vec{\phi}_1 \quad \vec{\phi}_2] = \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} \longrightarrow \text{Modal matrix}$$



# Introduction to Mode Superposition



- Let's define a set of *principal coordinates*,  $\eta_1(t)$  and  $\eta_2(t)$ .
- Then, by the mode superposition, the displacement response can be written as

$$u(t) = \vec{\phi}_1 \eta_1(t) + \vec{\phi}_2 \eta_2(t)$$

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} \begin{Bmatrix} \eta_1(t) \\ \eta_2(t) \end{Bmatrix} = \begin{Bmatrix} \eta_1(t) + \eta_2(t) \\ \eta_1(t) - 0.5\eta_2(t) \end{Bmatrix}$$

- Note that

$$u(t) = \Phi \eta(t)$$

# Introduction to Mode Superposition



- Pre-multiplying  $\Phi^T$  to the equation of motion,

$$\Phi^T [\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}(t)]$$

$$\Phi^T \mathbf{m} \Phi \ddot{\boldsymbol{\eta}}(t) + \Phi^T \mathbf{k} \Phi \boldsymbol{\eta}(t) = \Phi^T \mathbf{p}(t)$$

$$\mathbf{M} \ddot{\boldsymbol{\eta}}(t) + \mathbf{K} \boldsymbol{\eta}(t) = \mathbf{P}(t)$$

$$\mathbf{M} = \Phi^T \mathbf{m} \Phi$$

Modal mass

$$\mathbf{K} = \Phi^T \mathbf{k} \Phi$$

Modal stiffness

$$\mathbf{P}(t) = \Phi^T \mathbf{p}(t)$$

Modal force

$$\mathbf{M} = \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} = m \begin{bmatrix} 3 & 0 \\ 0 & -3/2 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} \begin{bmatrix} 2k & -k \\ -k & 3k \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} = k \begin{bmatrix} 3 & 0 \\ 0 & 15/4 \end{bmatrix}$$

$$\mathbf{P}(t) = \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} \begin{Bmatrix} P_1 \\ 0 \end{Bmatrix} \cos \Omega t = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \cos \Omega t$$

$$m \begin{bmatrix} 3 & 0 \\ 0 & 3/2 \end{bmatrix} \begin{Bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{Bmatrix} + k \begin{bmatrix} 3 & 0 \\ 0 & 15/4 \end{bmatrix} \begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_1 \end{Bmatrix} \cos \Omega t$$

$$3m\ddot{\eta}_1 + 3k\eta_1 = P_1 \cos \Omega t$$

$$(3/2)m\ddot{\eta}_2 + (15/4)k\eta_2 = P_1 \cos \Omega t$$

—————> Decoupled equation of motion  
Because of orthogonality relationship

- Assuming harmonic motion and solving the decoupled equations of motion,

$$\eta_1(t) = Y_1 \cos(\Omega t)$$

$$\eta_2(t) = Y_2 \cos(\Omega t)$$

- Substituting these into the decoupled equations of motion

$$Y_1 = \frac{P_1}{3k - 3m\Omega^2} \quad Y_2 = \frac{P_1}{(15/4)k - (3/2)m\Omega^2}$$

## □ Recalling

$$\mathbf{u}(t) = \Phi \boldsymbol{\eta}(t) = \begin{Bmatrix} \eta_1 + \eta_2 \\ \eta_1 - (1/2)\eta_2 \end{Bmatrix} \quad \begin{aligned} u_1(t) &= U_1 \cos \Omega t \\ u_2(t) &= U_2 \cos \Omega t \end{aligned}$$

$$U_1 = \frac{P_1/3k}{1 - (\Omega/\omega_1)^2} + \frac{4P_1/15k}{1 - (\Omega/\omega_2)^2}$$

$$U_2 = \frac{P_1/3k}{1 - (\Omega/\omega_1)^2} - \frac{1}{2} \left[ \frac{4P_1/15k}{1 - (\Omega/\omega_2)^2} \right]$$

## □ That is,

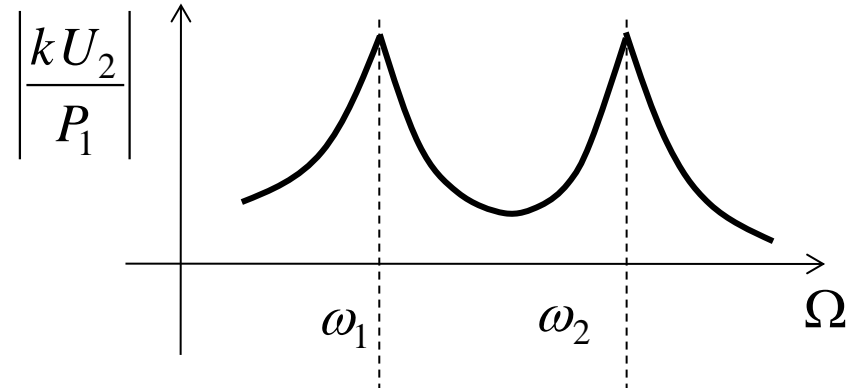
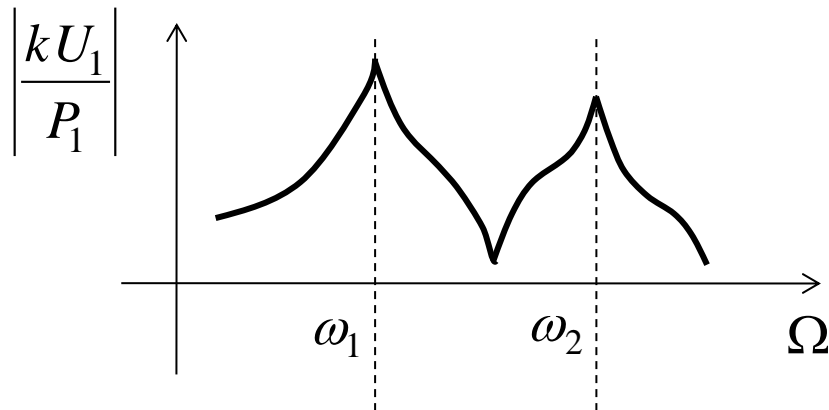
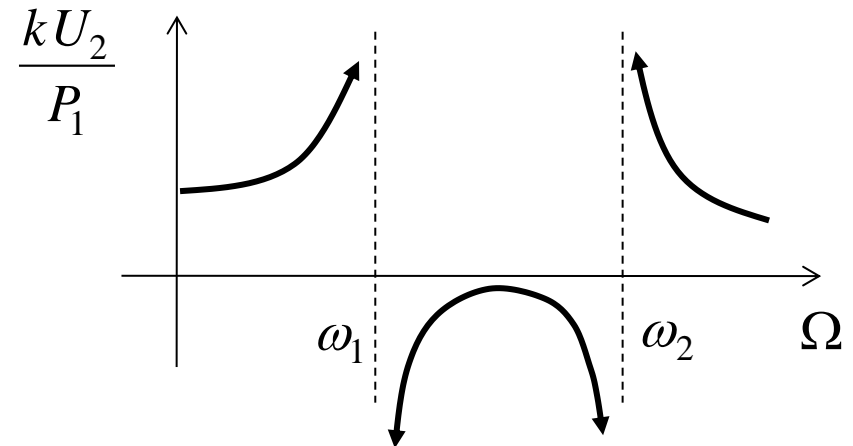
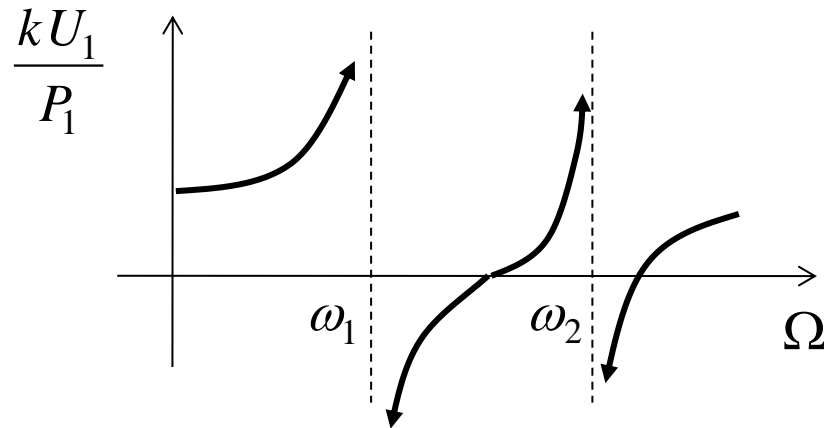
$$U_1 = \phi_{11}Y_1 + \phi_{12}Y_2$$

$$U_2 = \phi_{21}Y_1 + \phi_{22}Y_2$$

# Introduction to Mode Superposition



- Visualizing the response under harmonic excitation,



A large orange circle with a white number 4 inside, serving as a section marker.

# 4

## Vibration Properties of MDOF Systems: Modes and Frequencies

---

## □ Eigensolution

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = 0$$

$$\mathbf{u}(t) = \mathbf{U} \cos(\omega t - \alpha)$$

$$[\mathbf{K} - \omega^2 \mathbf{M}] \mathbf{U} = 0$$

$$\det[\mathbf{K} - \omega^2 \mathbf{M}] = 0 \longrightarrow \text{Characteristic equation}$$

## □ Solve for $\omega$

$$0 \leq \omega_1^2 \leq \omega_2^2 \leq \dots \leq \omega_N^2$$

## □ Scaling (normalizing) the modes

- Scale the r-th mode so that  $(\phi_i)_r = 1$  at a specified coordinate
- Scale the r-th mode so that  $(\phi_i)_r = 1$  where  $|(\phi_i)_r| = \max_j |(\phi_i)_j|$
- Scale the r-th mode so that  $M_r = \phi_r^T \mathbf{m} \phi_r = 1$

$$\phi_r^T \mathbf{k} \phi_r = K_r \quad \text{modal stiffness}$$

$$\phi_r^T \mathbf{k} \phi_r = \omega_r^2 (\phi_r^T \mathbf{m} \phi_r)$$

$$\omega_r^2 = \frac{K_r}{M_r}$$

## □ Mode shapes for distinct frequency cases

- Let's define the dynamic stiffness as

$$\mathbf{D}(\omega_r) \equiv \mathbf{k} - \omega_r^2 \mathbf{m}$$

- If the coordinate 1 is not a node point of the r-th mode

$$\begin{bmatrix} D_{aa}(\omega_r) & \mathbf{D}_{ab}(\omega_r) \\ \mathbf{D}_{ba}(\omega_r) & \mathbf{D}_{bb}(\omega_r) \end{bmatrix} \begin{Bmatrix} 1 \\ \{\boldsymbol{\phi}_b\}_r \end{Bmatrix} = \begin{Bmatrix} 1 \\ \mathbf{0}_b \end{Bmatrix}$$

$$\{\boldsymbol{\phi}_b\}_r = -[\mathbf{D}_{bb}(\omega_r)]^{-1} \mathbf{D}_{ba}(\omega_r)$$

$$\{\boldsymbol{\phi}_b\}_r = \begin{Bmatrix} \phi_2 \\ \phi_3 \\ \cdot \\ \cdot \\ \phi_N \end{Bmatrix}_r$$



## □ Orthogonality

- For the r-th mode, pre-multiplying  $\phi_s^T$

$$\phi_s^T \mathbf{k} \phi_r - \omega_r^2 (\phi_s^T \mathbf{m} \phi_r) = 0$$

- For the s-th mode, pre-multiplying  $\phi_r^T$

$$\phi_r^T \mathbf{k} \phi_s - \omega_s^2 (\phi_r^T \mathbf{m} \phi_s) = 0 \quad \text{-----} \quad (2)$$

$\mathbf{k}, \mathbf{m}$ : symmetric

$$\phi_s^T \mathbf{k} \phi_r - \omega_s^2 (\phi_s^T \mathbf{m} \phi_r) = 0 \quad \text{-----} \quad (3)$$

- Subtracting equation (3) from equation (2)

$$(\omega_r^2 - \omega_s^2)(\phi_s^T \mathbf{m} \phi_r) = 0$$

- Thus,

$\phi_s^T \mathbf{m} \phi_r = 0 \quad \text{if} \quad \omega_r \neq \omega_s$
---

r-th & s-th modes are orthogonal w.r.t. the mass matrix

# Vibration Properties of MDOF System



- From Equation (2),

$$\phi_s^T \mathbf{k} \phi_r = 0 \quad \text{if} \quad \omega_r \neq \omega_s$$

r-th & s-th modes are orthogonal w.r.t. the stiffness matrix

- For N-DOF system, the modal matrix is written as

$$\mathbf{\Phi} = \begin{bmatrix} \vec{\phi}_1 & \vec{\phi}_2 & \cdot & \cdot & \vec{\phi}_N \end{bmatrix}$$

- Then, algebraic equation can be written for all N modes

$$\mathbf{K}\mathbf{\Phi} = \mathbf{M}\mathbf{\Phi}\mathbf{\Lambda}$$

- where  $\mathbf{\Lambda} = \text{diag}(\omega_1^2, \omega_2^2, \omega_3^2, \dots, \omega_N^2)$   $\mathbf{M} = \mathbf{\Phi}^T \mathbf{m} \mathbf{\Phi} = \text{diag}(M_1, M_2, \dots, M_N)$   
 $\mathbf{K} = \mathbf{\Phi}^T \mathbf{k} \mathbf{\Phi} = \text{diag}(K_1, K_2, \dots, K_N)$

- If the natural modes are normalized so that  $M_r=1$ ,  $r=1,2,\dots,N$ , then

$$\Phi^T \mathbf{m} \Phi = \mathbf{I}$$

$$\Phi^T \mathbf{k} \Phi = \Lambda$$

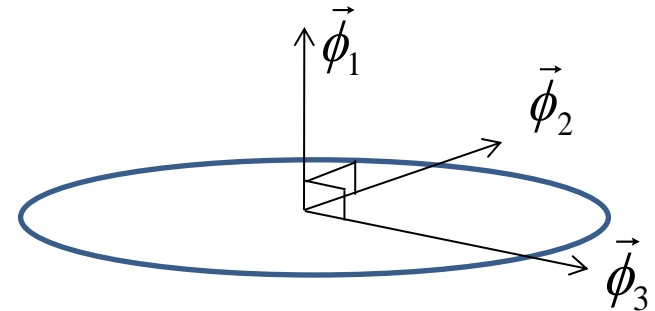
- **Mode shapes for repeated frequency cases**

$$\omega_r = \omega_{r+1} = \dots = \omega_{r+p-1}$$

- If the eigenvalues are repeated  $p$  times, then there could be  $(N-p)$  linearly dependent eigenvectors at least.
- For example, if  $\vec{\phi}_2$  and  $\vec{\phi}_3$  are eigenvectors corresponding to the repeated eigenvalues, they do not need to be orthogonal to each other. But they can be determined so that they are orthogonal to each other and the following always is satisfied.

$$\vec{\phi}_1 \perp \vec{\phi}_2 \quad \vec{\phi}_1 \perp \vec{\phi}_3$$

- So for this case, at least  $\vec{\phi}_1$  is the only one that is linearly independent.



# Vibration Properties of MDOF System



- The rank of  $D(\omega_r)$  will be  $N-p$  if  $\omega_r$  is repeated  $p$  times

$$\begin{bmatrix} D_{aa}(\omega_r)_{p \times p} & \mathbf{D}_{ab}(\omega_r)_{p \times (N-p)} \\ \mathbf{D}_{ba}(\omega_r)_{(N-p) \times p} & \mathbf{D}_{bb}(\omega_r)_{(N-p) \times (N-p)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\phi}_a \\ \boldsymbol{\phi}_b \end{Bmatrix}_r = \begin{Bmatrix} \mathbf{0}_a \\ \mathbf{0}_b \end{Bmatrix}$$

$$\{\boldsymbol{\phi}_a\} = \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_p \end{Bmatrix}_r \quad \{\boldsymbol{\phi}_b\} = \begin{Bmatrix} \phi_{p+1} \\ \phi_{p+2} \\ \vdots \\ \phi_N \end{Bmatrix}_r$$

$$\{\boldsymbol{\phi}_b\}_r = -[\mathbf{D}_{bb}(\omega_r)]^{-1} \mathbf{D}_{ba}(\omega_r) \{\boldsymbol{\phi}_a\}_r$$

# Vibration Properties of MDOF System



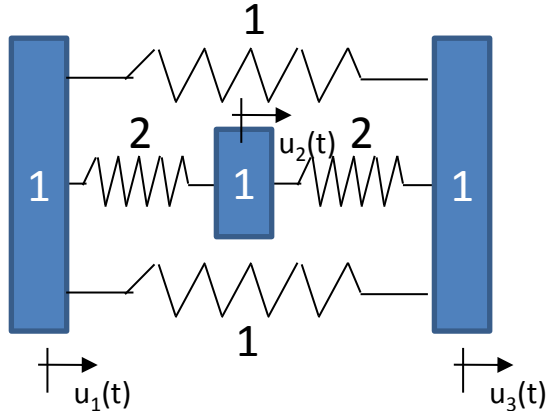
- Determine  $p$  vectors so that  $\{\boldsymbol{\varphi}\}_r, \{\boldsymbol{\varphi}\}_{r+1}, \dots, \{\boldsymbol{\varphi}\}_{r+p-1}$  are orthogonal to each other

$$\{\boldsymbol{\varphi}_a\}_r = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \quad \{\boldsymbol{\varphi}_a\}_{(r+1)} = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \quad \dots \quad \{\boldsymbol{\varphi}_a\}_{(r+p-1)} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{Bmatrix}$$

# Exam III (1)



- Consider the following assembly of masses and springs (numbers are mass and stiffness values)



- Then the equation of motion is as follows

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

- The dynamic matrix is obtained as

$$\mathbf{D}(\omega) = \begin{bmatrix} 4 - \omega^2 & -2 & -2 \\ -2 & 4 - \omega^2 & -2 \\ -2 & -2 & 4 - \omega^2 \end{bmatrix}$$

$$|\mathbf{D}(\omega)| = 0$$

$$\omega^2(\omega^4 - 12\omega^2 + 36) = 0$$

$$\omega_1^2 = 0 \quad \text{and} \quad \omega_2^2 = \omega_3^2 = 6$$

Corresponding to rigid body mode

□ For  $\omega_2^2 = \omega_3^2 = 6$

$$\left[ \begin{array}{cc|c} -2 & -2 & -2 \\ -2 & -2 & -2 \\ -2 & -2 & -2 \end{array} \right] \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Rank is 1

$$\{\boldsymbol{\phi}_b\}_r = -[\mathbf{D}_{bb}(\omega_r)]^{-1} \mathbf{D}_{ba}(\omega_r) \{\boldsymbol{\phi}_a\}_r$$

$$= -\left(-\frac{1}{2}\right) \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}_r = -(\phi_1)_r - (\phi_2)_r$$

□ Letting

$$\begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}_2 = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}_3 = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

$$(\phi_3)_2 = -1 \quad \text{and} \quad (\phi_3)_3 = -1$$

□ So the two modes corresponding to the repeated natural frequencies

$$\{\phi\}_2 = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} \quad \{\phi\}_3 = \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix}$$

□ For  $\omega_1^2 = 0$

$$\left[ \begin{array}{c|cc} 4 & -2 & -2 \\ \hline -2 & 4 & -2 \\ -2 & -2 & 4 \end{array} \right] \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix}_1 = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$



$$\{\boldsymbol{\phi}_b\}_1 = -[\mathbf{D}_{bb}(\omega_r)]^{-1} \mathbf{D}_{ba}(\omega_r) = -\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}^{-1} \begin{Bmatrix} -2 \\ -2 \end{Bmatrix}$$

$$= -\frac{1}{16-4} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \begin{Bmatrix} -2 \\ -2 \end{Bmatrix} = -\frac{1}{12} \begin{Bmatrix} -12 \\ -12 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\{\boldsymbol{\phi}\}_1 = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \quad \{\boldsymbol{\phi}\}_2 = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} \quad \{\boldsymbol{\phi}\}_3 = \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix}$$

□ Checking the mass-orthogonality of  $\{\boldsymbol{\phi}\}_2$  and  $\{\boldsymbol{\phi}\}_3$

$$\boldsymbol{\phi}_2^T \mathbf{m} \boldsymbol{\phi}_3 = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix} = 1 \neq 0$$

## □ Mass normalization

$$m_1^* = \phi_1^T \mathbf{m} \phi_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = 3$$

$$m_2^* = \phi_2^T \mathbf{m} \phi_2 = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} = 2$$

$$m_3^* = \phi_3^T \mathbf{m} \phi_3 = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix} = 2$$

- Divide each  $\{\phi\}_i$  by the square root of the modal mass

$$\{\phi\}_1^* = \frac{1}{\sqrt{3}} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{\phi\}_2^* = \frac{1}{\sqrt{2}} \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}$$

$$\{\phi\}_3^* = \frac{1}{\sqrt{2}} \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix}$$



# 5

## Frequency–Response Analysis of MDOF System

---

- Consider N-DOF system with viscous damping

$$\mathbf{C} = \mathbf{\Phi}^T \mathbf{c} \mathbf{\Phi} = \text{diag}(C_n) = \text{diag}(2\zeta_n \omega_n M_n)$$

- Then uncoupled modal equation of motion is as follows

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = \frac{1}{M_n} f_n(t) = \frac{1}{M_n} \mathbf{\Phi}^T \mathbf{p}(t)$$

$$n = 1, 2, \dots, N$$

$$\mathbf{p}(t) = \mathbf{P} e^{i\Omega t} \quad F_n = \phi_n^T \mathbf{P}$$

- Using the complex frequency response technique,

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = \omega_n^2 \frac{F_n}{K_n} e^{i\Omega t} \quad n = 1, 2, \dots, N$$

$$q_n = \bar{q}_n e^{i\Omega t} \quad \dot{q}_n = i\Omega \bar{q}_n e^{i\Omega t} \quad \ddot{q}_n = -\Omega^2 \bar{q}_n e^{i\Omega t}$$

$$\bar{q}_n(\omega_n^2 - \Omega^2 + i\Omega 2\zeta_n \omega_n) = \omega_n^2 \frac{F_n}{K_n}$$

$$\bar{q}_n = \frac{1/K_n}{(1-r_n^2) + i(2\zeta_n r_n)} F_n = \bar{H}_{q_n/F_n} F_n$$

$$q_n = \bar{H}_{q_n/F_n} F_n e^{i\Omega t}$$

$$\bar{q}_n = \frac{F_n / K_n}{(1-r_n^2) + i(2\zeta_n r_n)} e^{i\Omega t} = \frac{(F_n / K_n) e^{i\Omega t}}{\sqrt{(1-r_n^2)^2 + (2\zeta_n r_n)^2} e^{i\alpha}} \frac{(F_n / K_n)}{\sqrt{(1-r_n^2)^2 + (2\zeta_n r_n)^2}} e^{i(\Omega t - \alpha)}$$

$$\left[ \begin{array}{l} \text{Recall that } \bar{A} = A_R + iA_I \text{ in polar form} \\ \bar{A} = \sqrt{A_R^2 + A_I^2} e^{i\alpha} \quad \alpha = \tan^{-1} \frac{A_I}{A_R} \end{array} \right]$$

- Then the real part of  $q_n$

$$\text{Re}\{\bar{q}_n\} = \frac{(F_n / K_n)}{\sqrt{(1-r_n^2)^2 + (2\zeta_n r_n)^2}} \cos(\Omega t - \alpha)$$

$$\tan \alpha_n = \frac{2\zeta_n r_n}{1-r_n^2} \quad : \text{Phase angle}$$

- Then the complex FRF in physical coordinates  $\bar{\mathbf{u}}(t)$

$$\bar{\mathbf{u}}(t) = \mathbf{\Phi} \bar{\mathbf{q}}(t) = \sum_{n=1}^N \phi_n \bar{q}_n(t)$$

$$\bar{\mathbf{u}}(t) = \sum_{n=1}^N \frac{\phi_n \phi_n^T \mathbf{P}}{K_n} \frac{1}{(1-r_n^2) + i(2\zeta_n r_n)} e^{i\Omega t}$$

$\bar{H}_{ij}(\Omega)$  gives the response at  
Coordinate  $u_i$  to unit  
Harmonic excitation at  $p_j$

$$\bar{H}_{ij}(\Omega) = \bar{H}_{u_i / p_j}(\Omega) = \sum_{n=1}^N \frac{\phi_{in} \phi_{jn}}{K_n} \frac{1}{(1-r_n^2) + i(2\zeta_n r_n)}$$

Note that  $\phi_{in}$  is the element  
in row  $i$  of the  $n$ -th mode  $\phi_n$

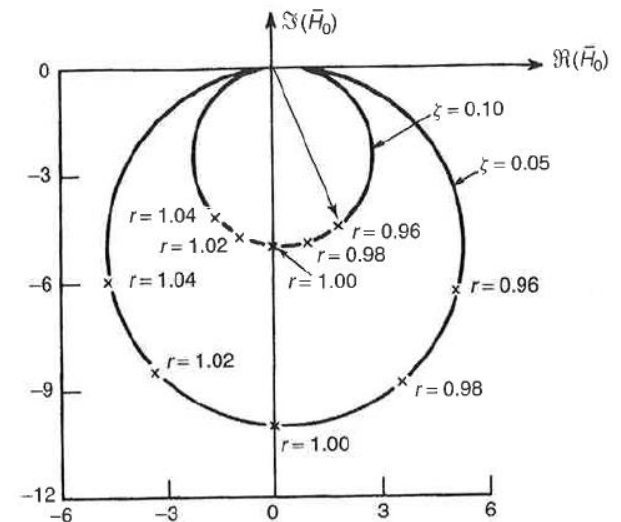
- Steady-state response  $\mathbf{u}(t)$  is obtained

$$\text{Re}\{\bar{\mathbf{u}}(t)\} = \sum_{n=1}^N \frac{\phi_n \phi_n^T \mathbf{P}}{K_n} \frac{1}{\sqrt{(1-r_n^2)^2 + (2\zeta_n r_n)^2}} \cos(\Omega t - \alpha)$$

- From the complex frequency response function, the Nyquist plot can be plotted

$$\text{Re}\{\bar{H}_{ij}(\Omega)\} = \sum_{n=1}^N \frac{\phi_{in} \phi_{jn}}{K_n} \frac{1-r_n^2}{(1-r_n^2)^2 + (2\zeta_n r_n)^2}$$

$$\text{Im}\{\bar{H}_{ij}(\Omega)\} = \sum_{n=1}^N \frac{\phi_{in} \phi_{jn}}{K_n} \frac{-2\zeta_n r_n}{(1-r_n^2)^2 + (2\zeta_n r_n)^2}$$





- The complete FRF will give the physical response of each coordinate to each input

$$\underbrace{\begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{Bmatrix}}_{\text{N inputs}} = \underbrace{\begin{bmatrix} \bar{H}_{11}(\Omega) & \bar{H}_{12}(\Omega) & \cdot & \cdot \\ \bar{H}_{21}(\Omega) & \bar{H}_{22}(\Omega) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \bar{H}_{NL}(\Omega) \end{bmatrix}}_{\substack{\text{Complete FRF or transfer function} \\ \text{which is symmetric!}}} \underbrace{\begin{Bmatrix} P_1 \\ P_1 \\ \vdots \\ P_L \end{Bmatrix}}_{\text{L inputs}}$$

A large orange circle with a white number 6 inside, serving as a section marker.

# 6

# Experimental Modal Testing

---

## ❑ **Excitation Methods**

### ❑ Based on wave forms

- ❑ Transient signals
- ❑ Random signals (white random, Pink random, Burst random)
- ❑ Periodic signals (sweep sine, pseudo-random, periodic chirp)

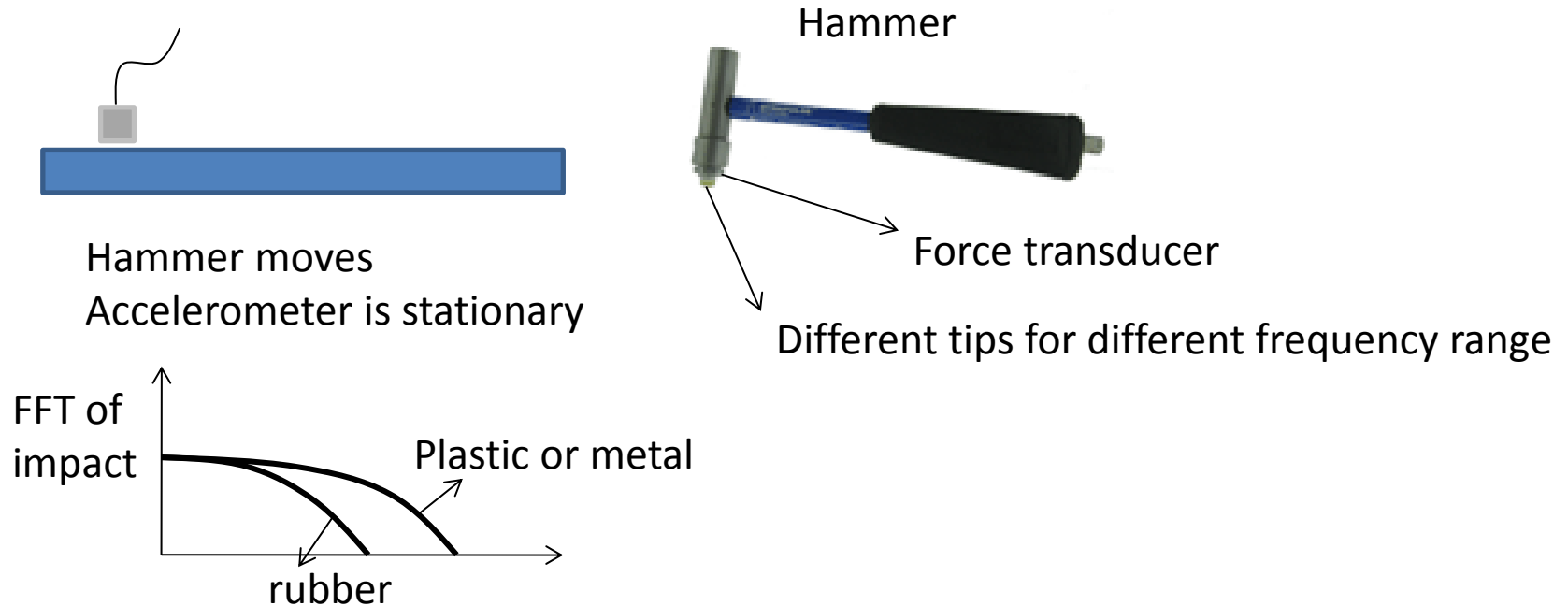
### ❑ Based on testing types

- ❑ Step-relaxation
- ❑ Impulse testing
- ❑ Shaker testing
- ❑ Test under operation

## ❑ **Characteristics of excitation methods**

### ❑ Impulse testing

- ❑ Using an impact hammer, it excites the structure. The ratio of Peak to RMS value is large. Thus, it is very likely to excite nonlinearity that the system has.
-

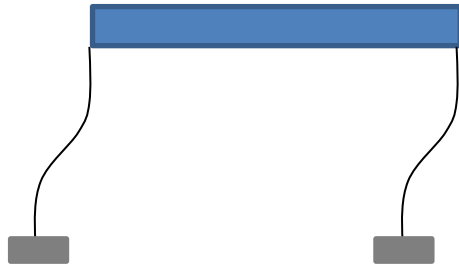


## ❑ Fast sine sweep testing

- ❑ Periodic chirp sweep frequency ranges multiple times. But
- ❑ But chirp signals do not repeat the sweep. Relatively, it needs less number of ensemble averaging.
- ❑ By narrowing the frequency range, it allows to observe the effect of nonlinearity.

- Step-relaxation testing

- It can be used for large structural systems.
- Energy in low frequency is large.



Pull a known distance and release

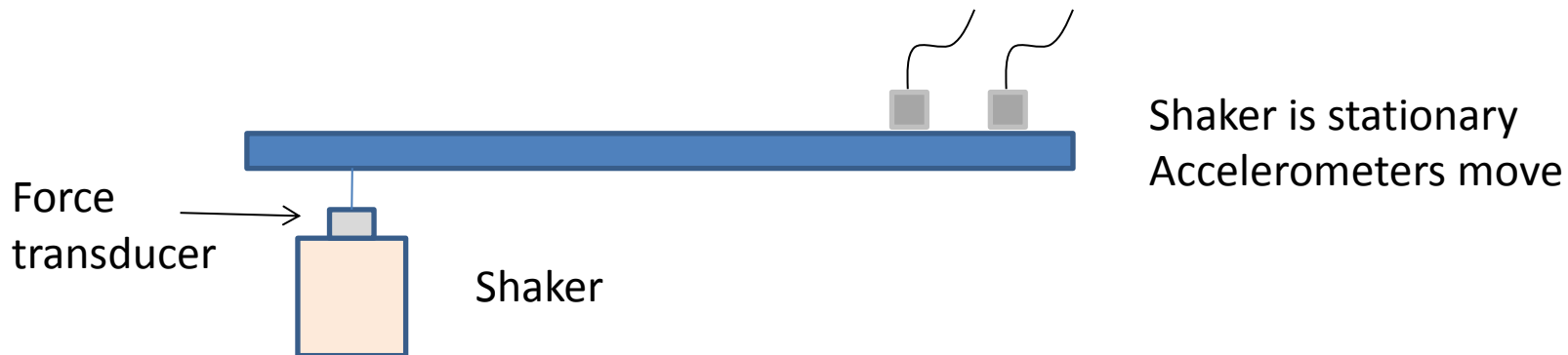
- Pure random

- Per each sampling, the measured signals are different. So it is suitable to remove the effect of nonlinearity, noise and measurement errors by taking a sufficient number of ensemble averaging.

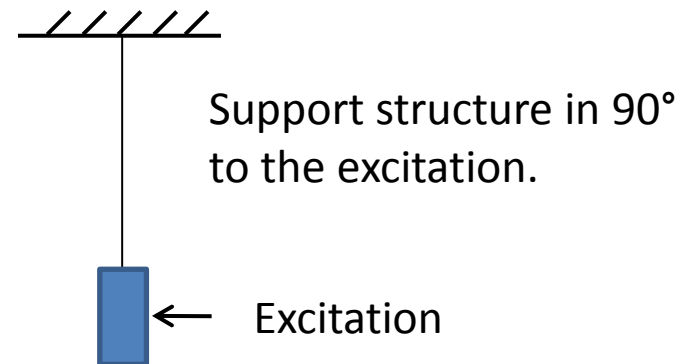
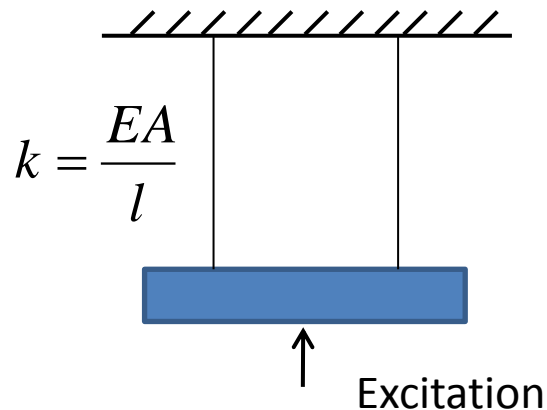
- Pseudo random

- Signals are generated in the frequency domain and converted to time domain signals. So, per each sampling cycle, the signal has periodicity. Unlike the pure random, the leakage is not a practical problem. Relatively, it needs a small number of ensemble averaging.

- Shaker testing



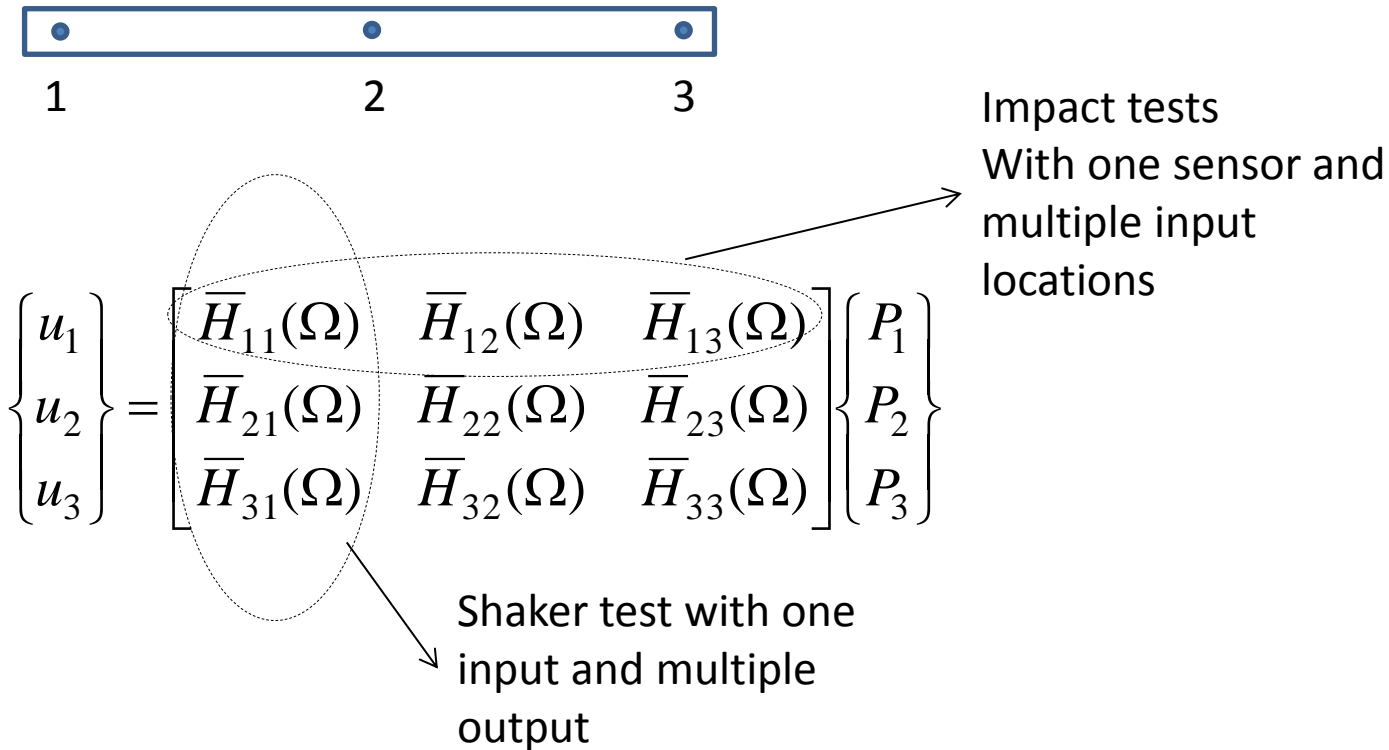
- ❑ **Methods of Supporting Structures:** Depending on the supporting method, quality of test results is determined since supporting methods change the boundary conditions and lead to dynamic properties of the structures.
  - ❑ Free-free supports
    - ❑ Under free-free supporting conditions, rigid body modes can also be detected. Under ideal free-free supporting conditions, the rigid body mode has 0 Hz. If stiff elastic supporting materials are used, the natural frequency of the rigid body mode increases and affects the vibrational modes in a negative way. Therefore, it is important to use soft elastic materials (for example, long elastic wires)



- ❑ Since large displacement due to rigid body motions can occur, non-contact sensors (i.e. laser vibrometer) may not be suitable for free-free supporting method.
  
  - ❑ Ground supports
    - ❑ It requires times and efforts.
    - ❑ Since the support should not move, the supporting part has to be stiff and massive.
    - ❑ External excitation force could not be measured. Therefore, it could be a source of errors.
-



- Mathematical difference between impact and shaker test



- Assumptions in experimental modal testing
  - Structures are linear elastic
  - Structures can be modeled as a finite DOF
  - Structures are viscously damped

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}e^{i\Omega t}$$

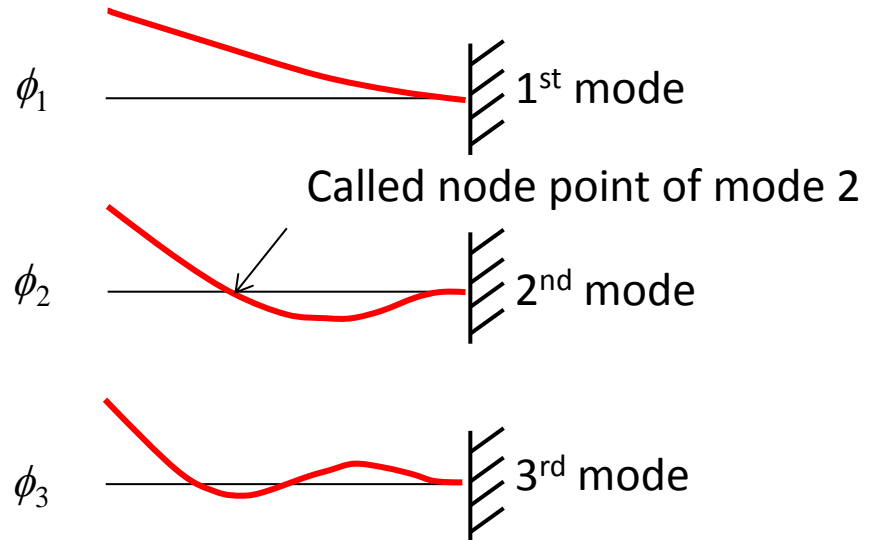
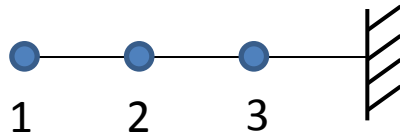
- Recalling the mode superposition representation of the frequency response function, the steady-state response  $\mathbf{u}(t)$  is as follows

$$\bar{\mathbf{u}}(t) = \sum_{n=1}^N \frac{\phi_n \phi_n^T \mathbf{P}}{K_n} \frac{1}{(1-r_n^2) + i(2\zeta_n r_n)} e^{i\Omega t}$$

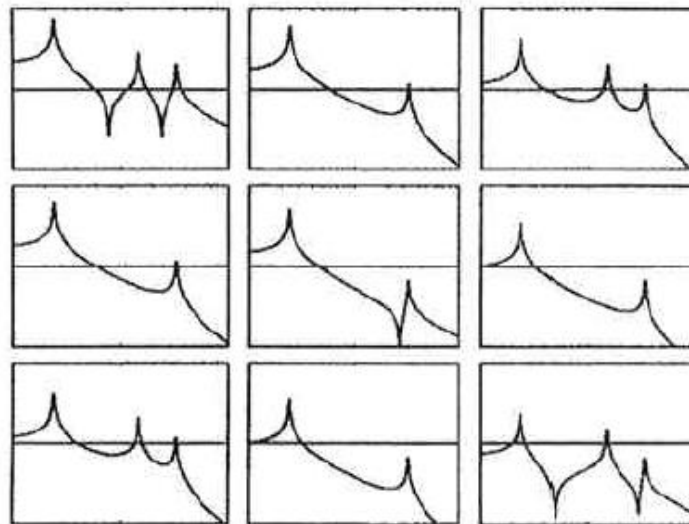
$$\bar{H}_{ij}(\Omega) = \bar{H}_{u_i / P_j}(\Omega) = \sum_{n=1}^N \frac{\phi_{in} \phi_{jn}}{K_n} \frac{1}{(1-r_n^2) + i(2\zeta_n r_n)}$$

Response for the response at  $i$  due to excitation at  $j$

# Experimental Modal Testing



$$\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix}$$

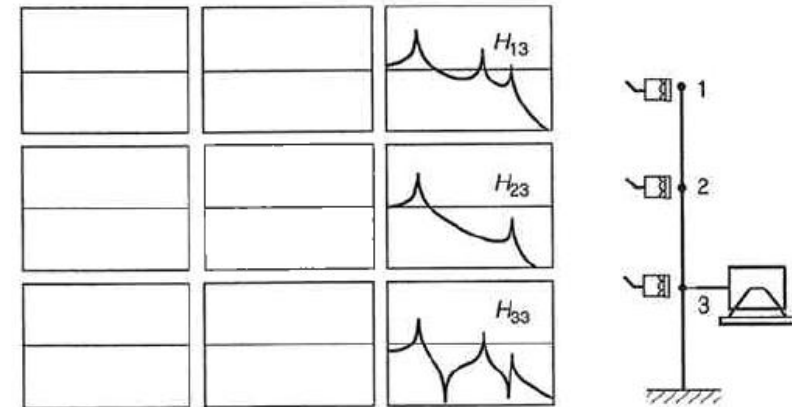
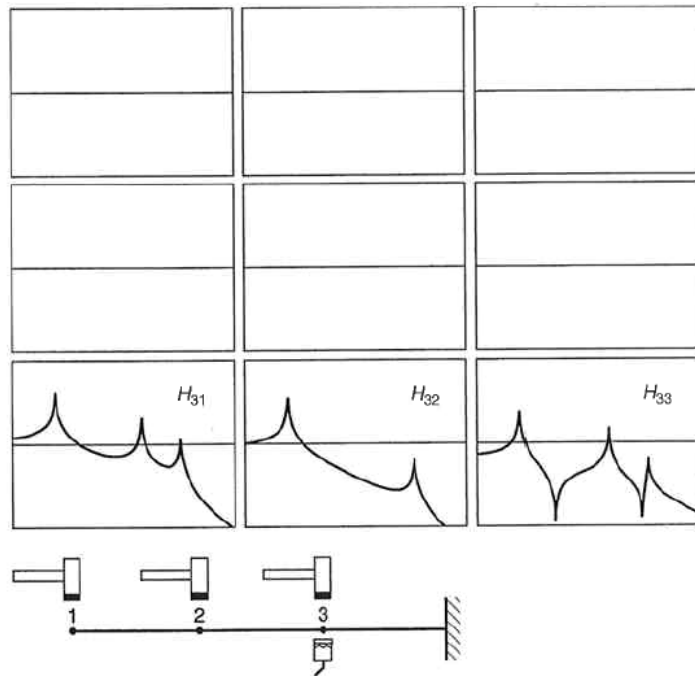


Receptance FRF magnitude  
Plots for an idealized 3DOF  
system

# Experimental Modal Testing



Measurement of a row of the  
FRF matrix  
Multiple-input-single-output (MISO)  
test



Measurement of a column of the  
FRF matrix  
Single-input-multiple-output (SIMO)  
test

## □ Frequency response function for SDOF system

### □ Receptance FRF

$$H_{u/p}(\Omega) = \frac{1/k}{(1-r^2) + i(2\zeta r)}$$

### □ Mobility FRF

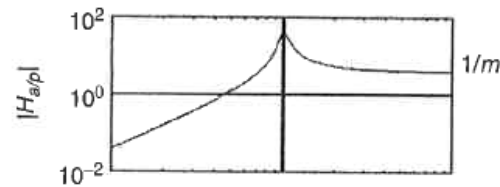
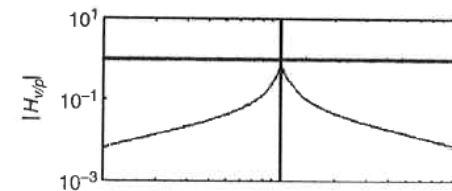
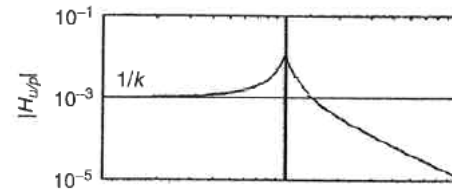
$$H_{v/p}(\Omega) = i\Omega \frac{1/k}{(1-r^2) + i(2\zeta r)}$$

### □ Accelerance FRF

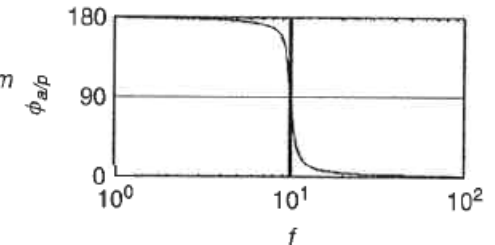
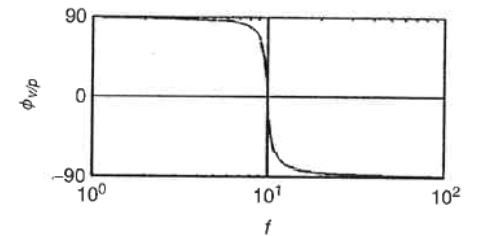
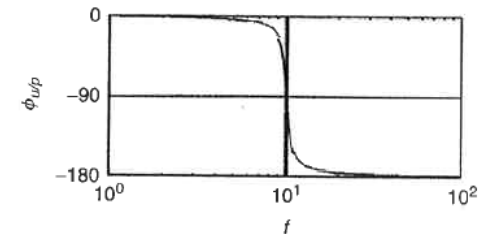
$$H_{a/p}(\Omega) = -\Omega^2 \frac{1/k}{(1-r^2) + i(2\zeta r)}$$

□ Note that the FRFs are complex.

Magnitude of FRF



Phase Angle



- Expressing FRFs in partial-fraction format (i.e. pole-residue format)

$$H_{u/p}(\Omega) = \frac{A}{i\omega - \lambda_1} + \frac{A^*}{i\omega - \lambda_1^*}$$

- Where A and A\* are complex conjugate residues

$$\lambda_1, \lambda_1^* = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2} = -\zeta\omega_n \pm i\omega_d \quad (\text{poles})$$

- A and A\* are pure imaginary quantities

$$A, A^* = \mp i \frac{1}{2m\omega_d}$$

- **Frequency Response Functions for MDOF system**

$$\left| \bar{H}_{ij}(\Omega) \right| = \left| \bar{H}_{u_i/p_j}(\Omega) \right| = \sum_{n=1}^N \frac{|\phi_{in}\phi_{jn}|}{K_n} \frac{1}{\sqrt{(1-r_n^2)^2 + (2\zeta_n r_n)^2}} \quad \tan \alpha_n(\Omega) = \frac{2\zeta_n r_n}{1-r_n^2}$$

$$\bar{H}_{v_i / P_j}(\Omega) = i\Omega \sum_{n=1}^N \frac{\phi_{in}\phi_{jn}}{K_n} \frac{1}{(1-r_n^2) + i(2\zeta_n r_n)}$$

$$\bar{H}_{a_i / P_j}(\Omega) = -\Omega^2 \sum_{n=1}^N \frac{\phi_{in}\phi_{jn}}{K_n} \frac{1}{(1-r_n^2) + i(2\zeta_n r_n)}$$

- Transforming the receptance FRF to the partial fraction form

$$H_{u_i / p_j}(\Omega) = \sum_{n=1}^N \left( \frac{A_{ijn}}{i\omega - \lambda_1} + \frac{A_{ijn}^*}{i\omega - \lambda_1^*} \right)$$

- where  $A_{ijr}$  and  $A_{ijr}^*$  are complex conjugate modal residues

$$A_{ijn}, A_{ijn}^* = \mp i \frac{\phi_{in}\phi_{jn}}{2M_n\omega_{dn}}$$

- However, in testing we don't know  $M_n$ .

- Therefore,

$$H_{u_i / p_j}(\Omega) = \sum_{n=1}^N \left( \frac{Q_n \phi_{in} \phi_{jn}}{i\omega - \lambda_1} + \frac{Q_n^* \phi_{in} \phi_{jn}}{i\omega - \lambda_1^*} \right)$$

- $Q_n$  are determined by curve-fitting to the measured FRF.



- When the natural frequencies are widely separated and the forcing frequency is equal to one of the undamped natural frequencies,  $\Omega = \Omega_n$ , the FRFs are dominated by the corresponding n-th mode.

$$\bar{H}_{a_i/P_j}(\Omega) = -\Omega^2 \sum_{n=1}^N \frac{\phi_{in}\phi_{jn}}{K_n} \frac{1}{(1-r_n^2) + i(2\zeta_n r_n)}$$

- From this equation, when  $r_n = 1$

$$\bar{H}_{a_i/P_j}(\Omega = \Omega_n) \approx i \frac{\phi_{in}}{2K_n\zeta_n} \phi_{jn} = i \frac{\phi_{jn}}{2K_n\zeta_n} \phi_{in} \quad \text{-----} \quad (4)$$

- Therefore, mode shape n can be obtained by examining the imaginary part of the FRFs for any output measurement point i for which  $\phi_{in} \neq 0$ , while the structure is subjected to harmonic excitation at force frequency  $\Omega = \Omega_n$  at each DOF j. Thus, a row of the FRF matrix should be inspected.

# Quadrature Peak-Picking Method

---



- Or, mode shape  $n$  can also be obtained by examining the imaginary part of the FRFs for each output measurement point  $I$  while the structure is subjected to harmonic excitation at force frequency  $\Omega = \Omega_n$  at any DOF  $j$  for which  $\phi_{jn} \neq 0$ . Thus, a row of the FRF matrix should be inspected.

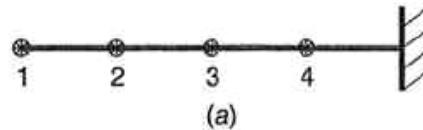
# Example IV



- Use the equation (4) to estimate the fundamental mode shape of the 4-DOF beam whose accelerance FRFs are shown as below.

- The first peak occur at 35 Hz.

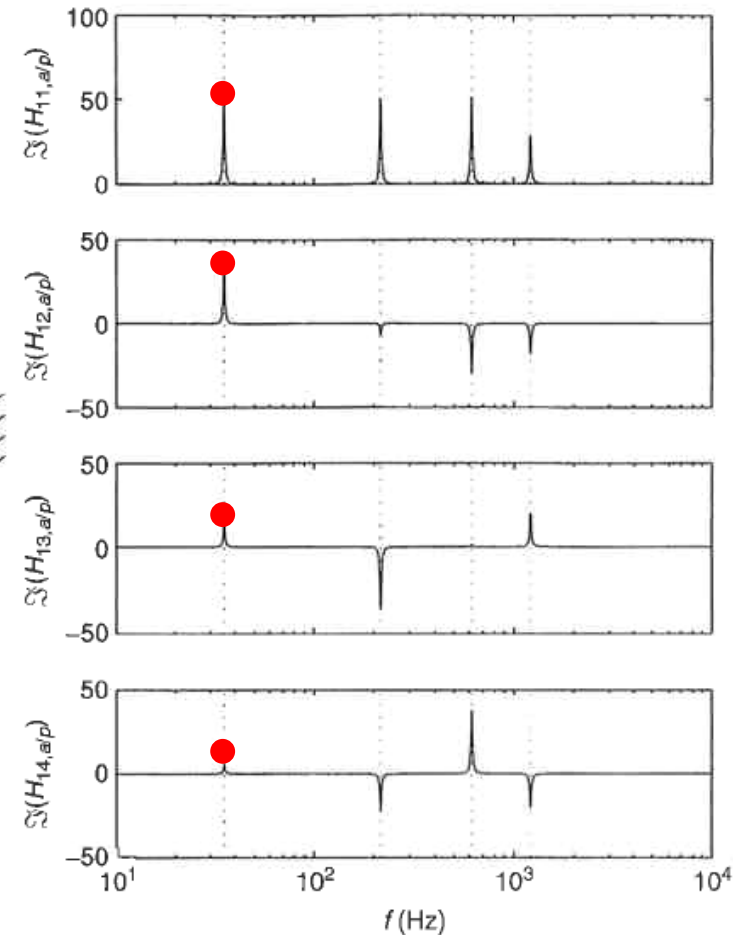
$$\phi_1 = [49.8 \quad 32.8 \quad 16.9 \quad 4.84]$$



- After normalizing,

$$\phi_1 = [1.0 \quad 0.66 \quad 0.34 \quad 0.10]$$

the first mode shape is obtained.



7

## Hands-on Experiment Project

# Problem Statement (1)

---

- ❑ **Conduct a fast sweep sine test** using a beam structure and measure dynamic response and input excitation signals.
  - ❑ **Determine natural frequencies and mode shapes from the measured FRFs** by the provided NI-LabVIEW VI program.
  - ❑ The tested beam is a fixed-fixed carbon steel beam.
-

# Guides for Reports

---

- ❑ Write a full report using the instructions provided in class. Organize your report into sections (e.g. Introduction, Procedures, Results, Discussion, Summary, References). Write concisely and clearly.
  - ❑ Include the following: (1) A schematic diagram and description of the test equipment. (2) Plots of the time-domain vibration response data measured from the experiment. (3) Plots of magnitudes, imaginary, real parts of the FRFs, and phase angles. (4) Plots of Nyquist and coherence. (5) Plot of mode shapes determined by the quadrature peak-picking method.
-

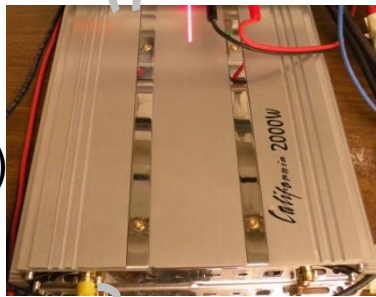
# 8

# Test Setup and Equipment

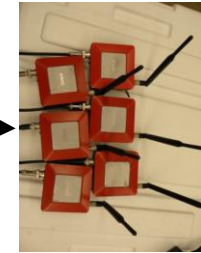
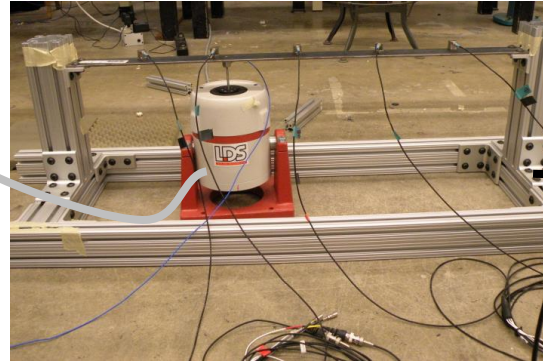
# Test Setup and Procedures (1)



Power Supplier(MPJA 14604PS)



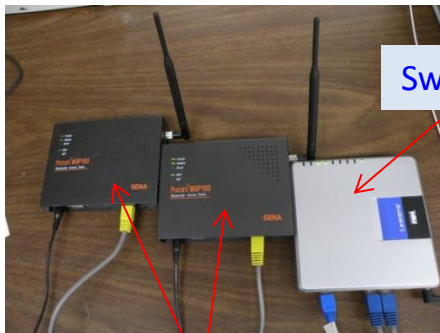
Amplifier(AP2000)



6 AnyLoggerS-V/ICP

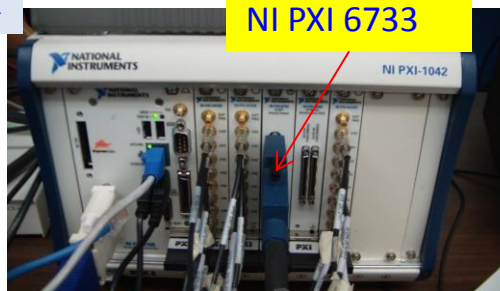


Connector Block (SCB68)



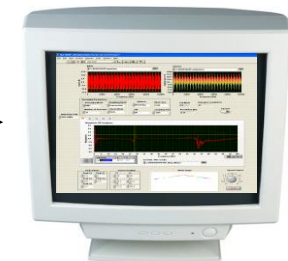
Switch box

MSP-100



NI PXI 6733

NI-PXI 8105 in NI-PXI 1042



NI- LabVIEW VI Program

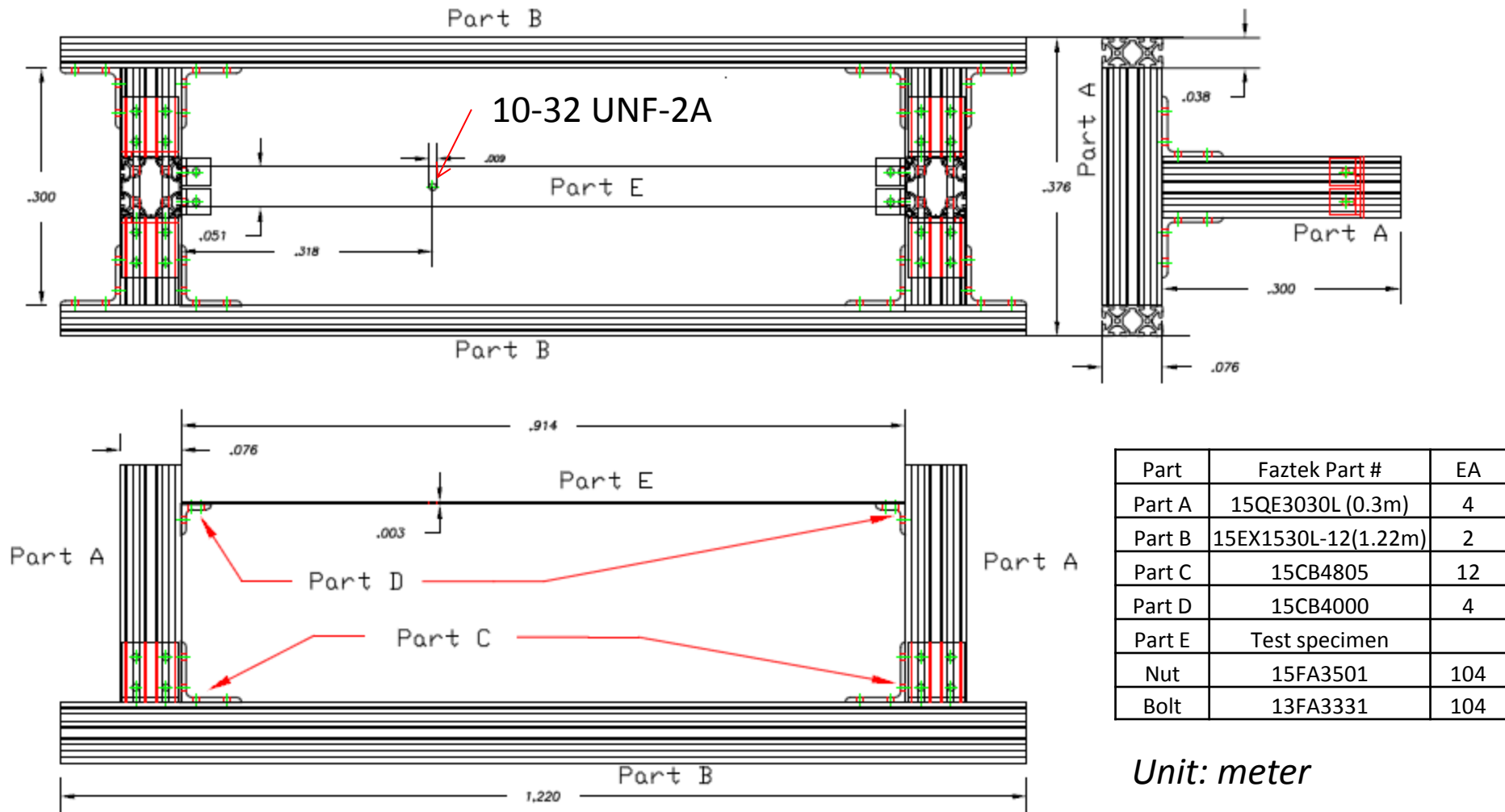


# Test Setup and Procedures (2)

---

- ❑ Step1 Turn on power supply.
- ❑ Step2 Generate excitation signals using NI-LabVIEW and send them through analog output board (NI-PXI 6733).
- ❑ Step3 Under the vibration excitation, proceed the test.

# Test Structure- Drawing



- ❑ Agilent 33250A (For function generator based test)
  - ❑ Amplifier (California AP2000 2000W)
  - ❑ Power Supply (MPJA 14604PS)
  - ❑ NI-PXI 8105 Controller
  - ❑ NI-PXI 6733 (For NI analog out signal generator based test)
  - ❑ 68-Pin Connector Block (SCB68)
  - ❑ ICP type Dytran triaxial accelerometer (3093B1)
  - ❑ ICP type PCB force sensor (208C02)
  - ❑ Six-channel AnyLogger (Korea Maintenance Co., LTD): AnyLoggerS-V/ICP and two MSP-100 modules
  - ❑ NI-LabVIEW 8.6
-

## Specification



- ☐ 320 watts RMS x 2 at 4 ohms
- ☐ 480 watts RMS x 2 at 2 ohms
- ☐ 960 watts RMS x 1 bridged output at 4 ohms
- ☐ 4-ohm stable in bridged mode
- ☐ Stereo or bridged mono output
- ☐ Tri-way capable (Tri-Way Crossover required)
- ☐ Dual power supply for stability at high volumes
- ☐ Fuse rating: 25A x 4
- ☐ Requires 4-gauge power and ground leads and a 100-amp fuse
- ☐ Wiring and hardware not included with amplifier
- ☐ Variable low-pass filters (50-250 Hz, 12 dB/octave)
- ☐ Variable bass boost (0-12 dB) at 45 Hz
- ☐ Variable subsonic filter (20-50 Hz)
- ☐ Preamp-level inputs (speaker-level to preamp-level adapter included)
- ☐ Preamp outputs
- ☐ Wired bass level remote control
- ☐ 24-1/4"W x 2"H x 10-1/4"D

## Specification



- ☐ Input voltage : 110-127 AC
- ☐ Output voltage : 0-30 DC
- ☐ Current : 0-10 A
- ☐ Voltage regulation
  - ☐ CV  $\square 1 \times 10^{-4} + 3 \text{mV}$
  - ☐ CC  $\square 2 \times 10^{-3} + 6 \text{mA}$
- ☐ Load regulation
  - ☐ CV  $\square 5 \times 10^{-4} + 3 \text{mV}$
  - ☐ CC  $\square 5 \times 10^{-4} + 6 \text{mA}$
- ☐ Ripple & noise
  - ☐ CV  $< 1.5 \text{mVrms}$
  - ☐ CC  $< 10 \text{mArms}$
- ☐ Protection : current limiting
- ☐ Voltage indication accuracy :  $1\% + 1 \text{d}$
- ☐ Current indication accuracy :  $1\% + 1 \text{d}$
- ☐ Ambient temperature :  $0 \sim 40^\circ\text{C}$
- ☐ Humidity:  $< 90\%$

## Specification

- ❑ Intel Core Duo Processor T2500(2.0 GHz dual core)
- ❑ 512 MB (1 x 512 MB DIMM) dual channel 667 MHz DDR2 RAM standard, 4 GB (2 x 2 GB DIMMs) maximum
- ❑ Integrated I/O
  - ❑ 10/100/1000BASE-TX Ethernet
  - ❑ 4 Hi-Speed USB ports
  - ❑ ExpressCard/34 slot
  - ❑ DVI-I video connector
  - ❑ GPIB (IEEE 488) controller
  - ❑ RS232 serial port
  - ❑ IEEE 1284 ECP/EPP parallel port
  - ❑ Integrated hard drive
- ❑ Internal PXI trigger bus routing
- ❑ Watchdog timer Software
- ❑ Hard drive-based recovery image PXI System



## Specification

- ❑ 8 high-speed digital I/O lines; two 24-bit counters; digital triggering
- ❑ Onboard or external update clock PXI trigger bus for synchronization with DAQ motion, and vision products
- ❑ NI DAQmx driver with configuration utility to simplify configuration and measurement
- ❑ Superior integration: LabVIEW, LabVIEW Real-Time, LabWindows™/CVI, and Measurement Studio for VB
- ❑ 1MS/s, 16-Bit, 8 Channels



# 68-Pin Connector Block (SCB68)

## Specification



- ☐ Number of channels : 8 differential, 16 single-ended
- ☐ Accuracy :  $\pm 1.0^{\circ}\text{C}$  over a  $0^{\circ}$  to  $110^{\circ}\text{C}$  range
- ☐ Output :  $10\text{ mV}/^{\circ}\text{C}$
- ☐ I/O connectors One 68-pin male SCSI connector
- ☐ Temperature :  $0^{\circ}$  to  $70^{\circ}\text{C}$
- ☐ Relative humidity : 5% to 90% non-condensing
- ☐ Temperature :  $-55^{\circ}$  to  $125^{\circ}\text{C}$
- ☐ Relative humidity : 5% to 90% non-condensing



# Dytran Accelerometer (3093B1)



Model 3093B1 Dytran Triaxial Accelerometer

Specification	Value	Unit
Weight	13.5	Grams
Size(Height x Width x Depth)	0.54 x 0.59 x 0.59	Inch
Sensitivity	100	mV/G
Ranges	+/-50	G
Frequency Response	2 to 5000	Hz
Equivalent Electrical Noise	0.007	G, RMS
Linearity	1	% F.S.
Temp. Range	-60 ~ +250	°F
Supply Current Range each axis	2 to 20	mA
Supply Voltage Range each axis	+18 to +30	VDC
Output impedance	100	OHMS

# Specification of Force Sensor



## Force Sensor (PCB 208C02)

Specification	Value	Unit
Weight	22.7	Grams
Size(Hex x Height x Sensing Surface)	15.88 x15.88 x 15.7	mm
Sensitivity	11241	mV/kN
Ranges	$\pm 0.4448$	kN
Linearity	Below 1	% F.S.
Temp. Range	-54 to +121	°C
Supply Current Range each axis	2 to 20	mA
Supply Voltage Range each axis	30	VDC
Output impedance	Below 100	OHMS



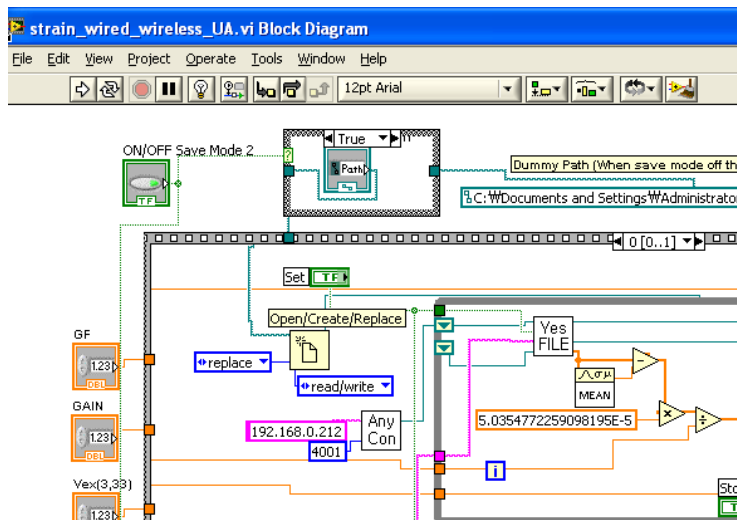
AnyloggerS-V/ICP for acceleration transmitter  
AnyLoggerS-B for strain transmitter

## Specification

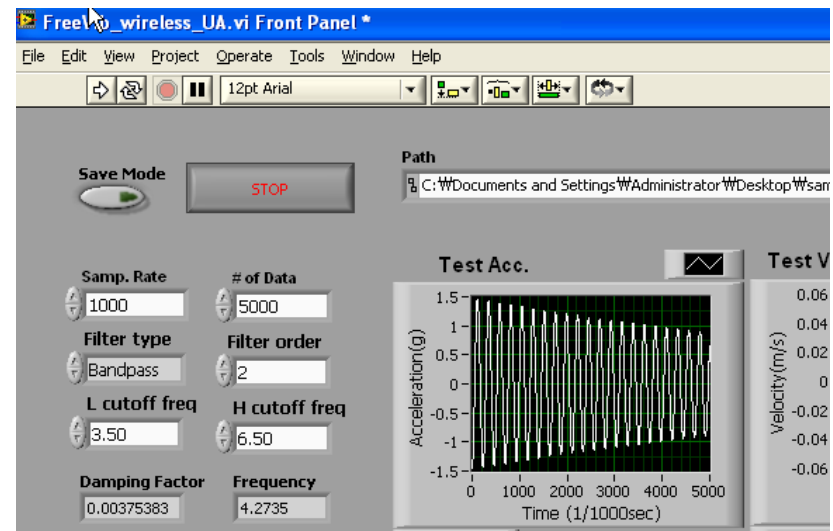
Contents	AnyloggerS-V/ICP	AnyLoggerS-B
<b>Support Num. of Channel</b>	1	1
<b>Input Voltage Range</b>	-5 ~ 5V	0 ~ 3V
<b>Gain</b>	1,2,5,10,20,50,100	50,100,250,500,1000,2500,5000
<b>Programmable</b>	10~1000Hz (10,20,50,100,200,500,1000)	10 ~ 1000Hz
<b>Lower Pass Filter</b>		10,20,50,100,200,500,1000
<b>Prog. Offset</b>	0 ~ 5V(12Bit)	0 ~ 3.3V(12Bit)
<b>Max Sampling Rate</b>	1000Hz	1000Hz
<b>Exciting Voltage</b>	24V(Only ICP Type)	3.3V±0.5%
<b>Connector</b>	BNC Connector	4Pin Circular Connector
<b>SIM Usage</b>	No	Yes
<b>Power Consumption (w/o sensor)</b>	150mA	100mA(w/o SIM)
<b>Internal Battery</b>	Li-ion Rechargeable 1500mAh x 2EA(Serial)	Li-ion Rechargeable 1500mAh x 2EA(Serial)
<b>Ext. Power Requirement</b>	5V	5V
<b>Operation Temperature</b>	-10 ~ 80℃	-10 ~ 80℃
<b>Sync. Accuracy</b>	< 10ms	< 10ms
<b>Dimension</b>	800 x 973 x 353	800 x 883 x 353
<b>Weight</b>	210g	210g
<b>ADC Resolution</b>	Differential 16Bit	Differential 16Bit
<b>Measurement Accuracy</b>	F.S. 0.1%	F.S. 0.1%
<b>Sensor Connectivity</b>	Voltage or ICP source	Bridge type sensor
<b>Signal Ripple</b>	Depends on Gain	Depends on Gain
<b>Communication</b>	Bluetooth v1.2 class1 18dBm(w/o ant.)	Bluetooth v1.2 class1 18dBm(w/o ant.)
<b>Radio Frequency Range</b>	2.402 ~ 2.480GHz	2.402 ~ 2.480GHz
<b>Transmission Method</b>	FHSS(freq. Hopping Spread Spectrum)	FHSS
<b>Modulation Method</b>	GFSK(Gaussian-filtered Freq. Shift Keying)	GFSK
<b>Approvals</b>	MIC, FCC, CE	MIC, FCC, CE

❑ LabVIEW (Laboratory Virtual Instrument Engineering Workbench) is a graphical programming language that uses icons instead of lines of text to create applications.

❑ In contrast to text-based programming languages, where instructions determine the order of program execution, LabVIEW uses dataflow programming, where the flow of data through the nodes on the block diagram determines the execution order of the VIs and functions. VIs, or virtual instrument, are LabVIEW programs that imitate physical instruments.



VI Block Diagram



Front Panel

9

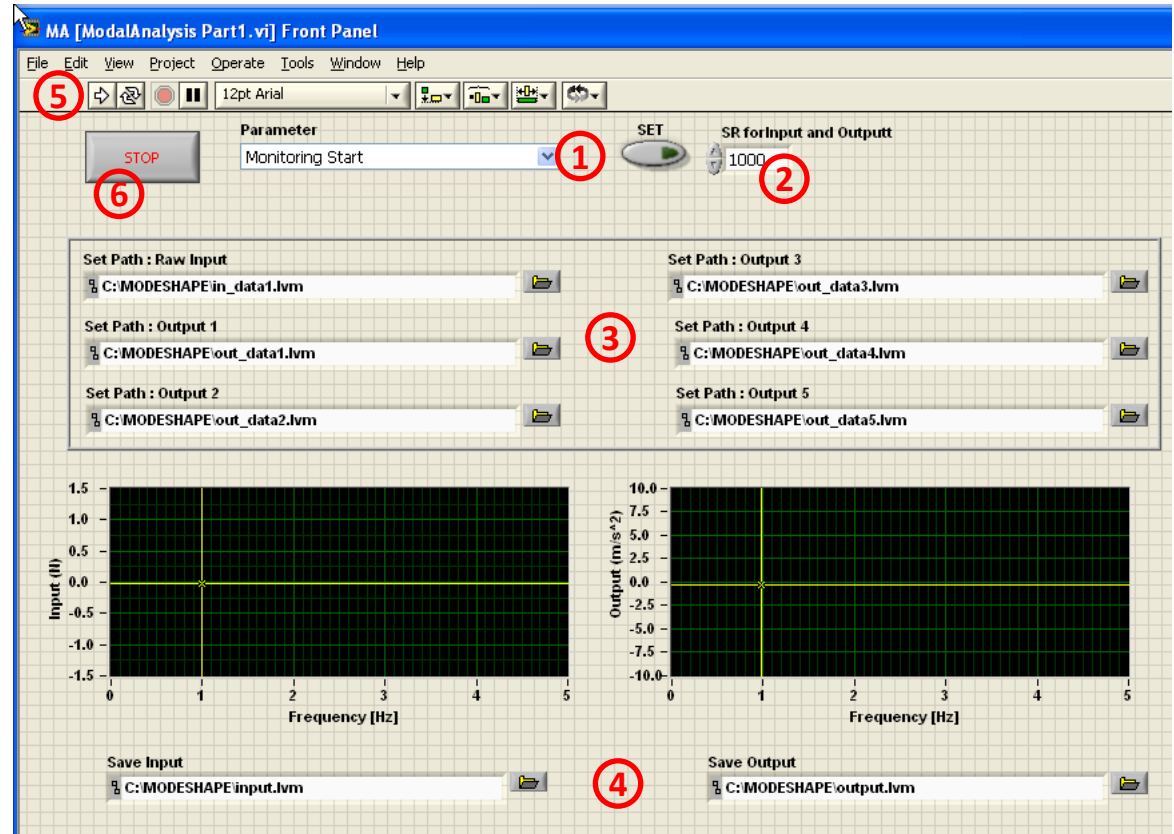
# Users Manual of NI-LabVIEW VI Program

---

## Modal Analysis PART 1.VI

Modal Analysis PART1.VI measures raw data and converts raw data to physical input forces and output accelerations and save.

- 1) **Set Parameter** – to “Monitoring Start” and click “Set”.
- 2) **Set Sampling Rate** – Default 1KHz for both input and output.
- 3) **Set Path** – Create empty files in “Path” where the raw data of input and outputs will be saved. One input and five outputs were used in this test.
- 4) **Set Path** – Create empty files in “Path” where the physical input force and output acc. data will be saved. Five outputs will be saved in one file.
- 5) **Run**- Run the front panel while the structure is vibrating.
- 6) **Stop** –Users can control the amount of data by running time.

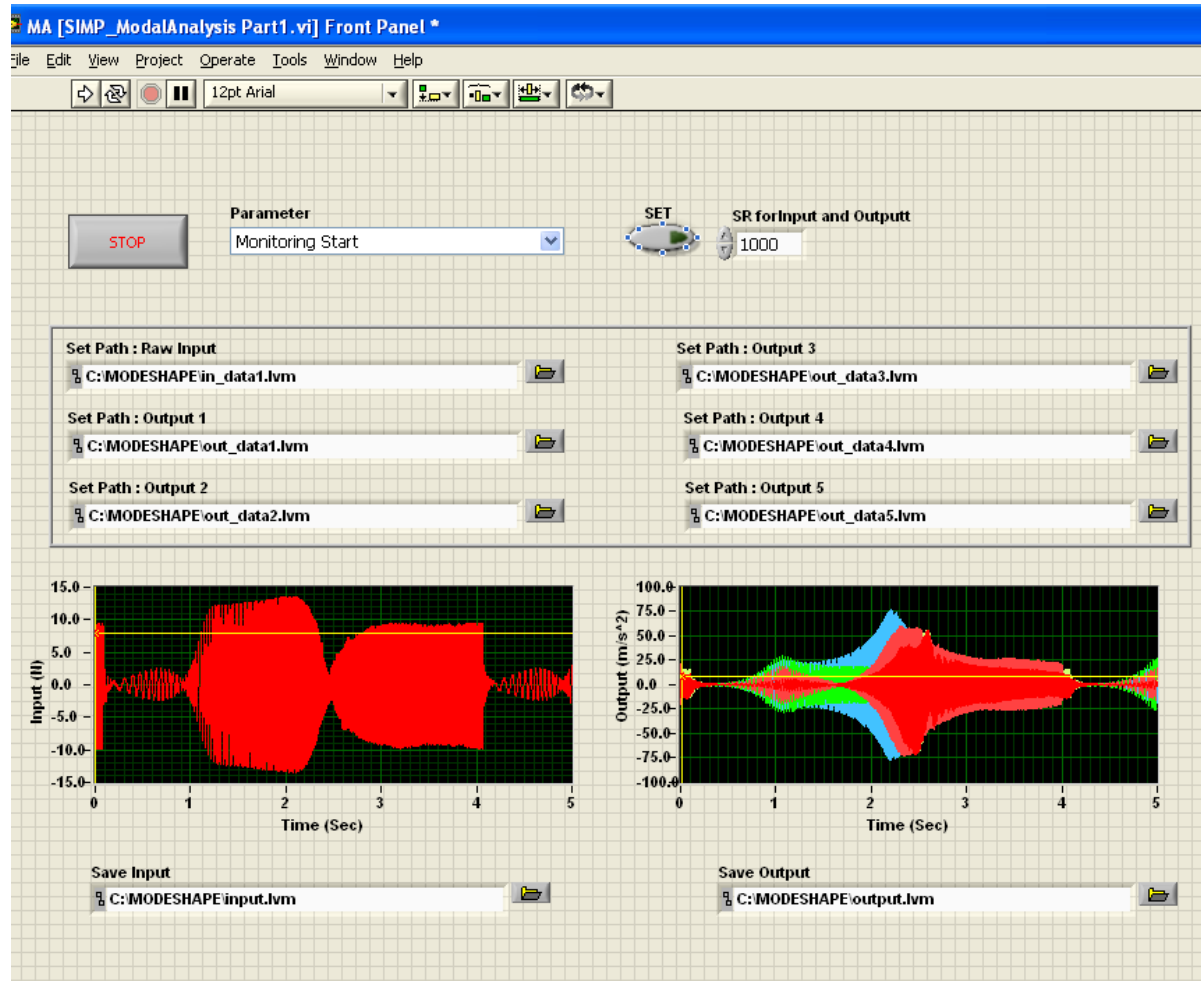


# Users Manual of LabVIEW VI Prog. (2)

## PART 1.VI

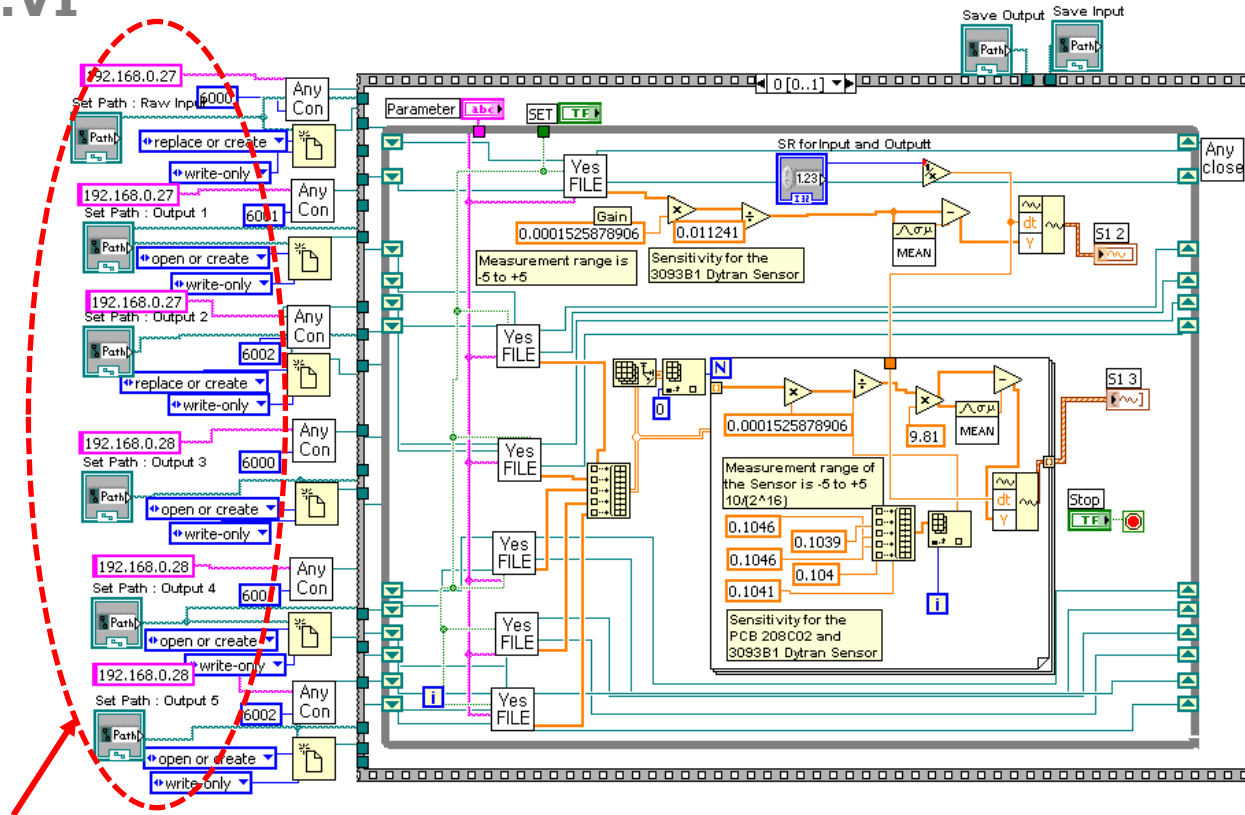
After acquiring raw data, the VI program automatically converts raw data to physical input forces and output accelerations and save them.

One input data are saved to the path of input and five output data are save to the path of output.





## PART 1.VI

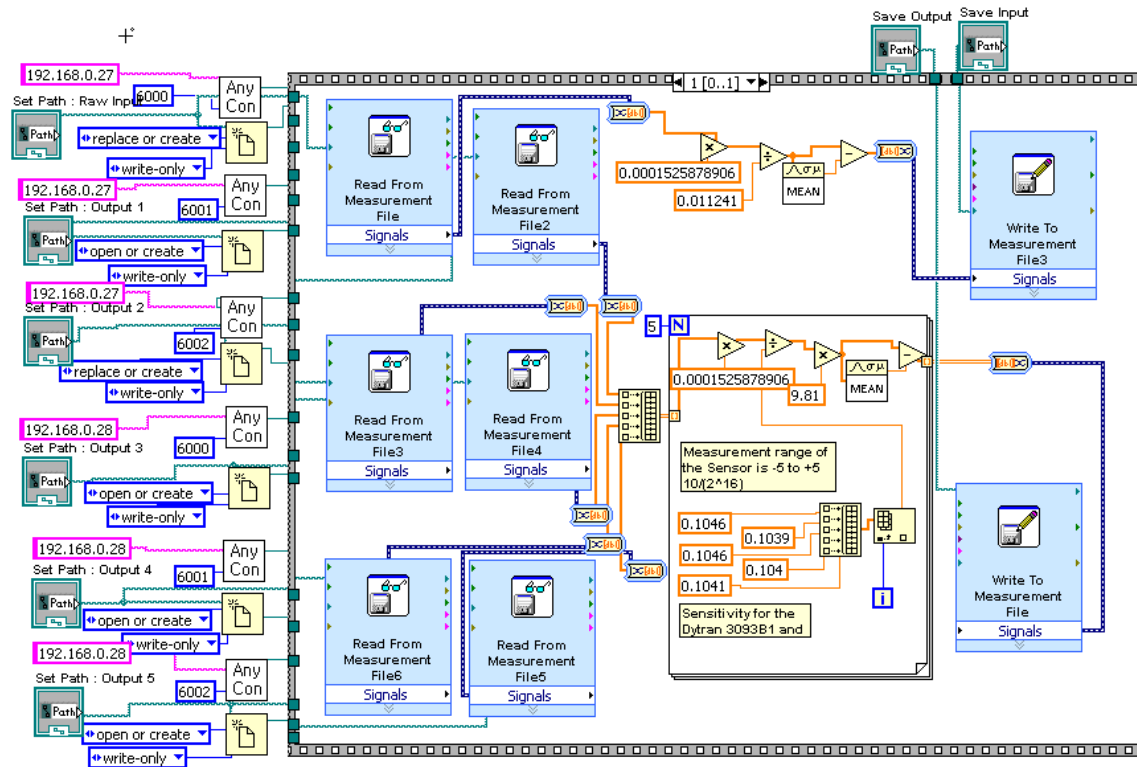


Set IP addresses and port numbers

The First Stack of Block Diagram of PART1.VI

- ❑ Two-stack sequences were used for Part1.VI.
- ❑ At the first sequence, the program saves the raw data to the file in the given path.
- ❑ Users should define correct **IP address** and **port number** based on the equipment following the AnyLogger® manual.

## PART 1.VI



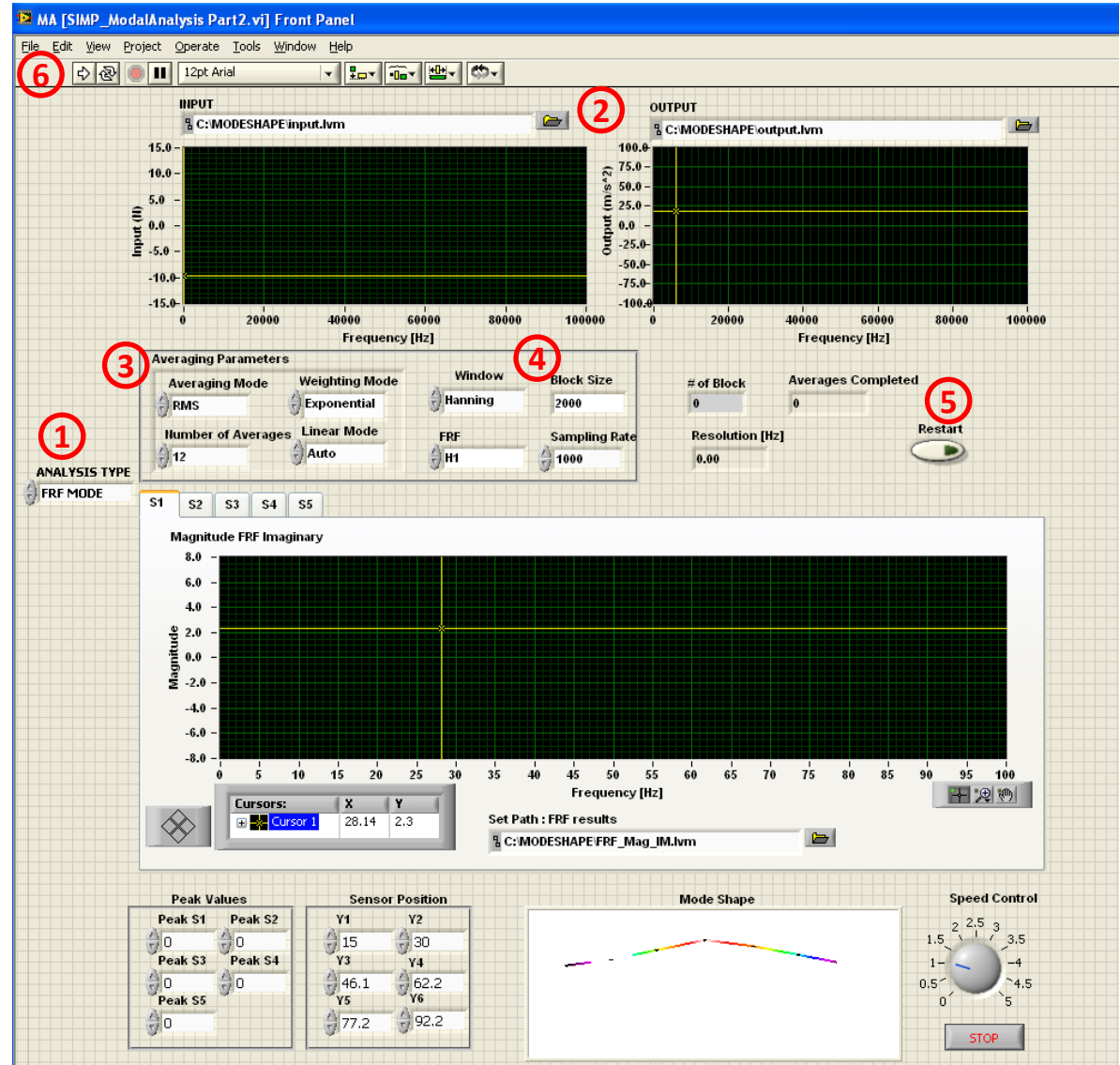
The Second Stack of Block Diagram of PART1.VI

- At the second sequence, the program reads the raw data from the files in the given paths and converts to physical input force and output acceleration data and save as data file.

## PART 2.VI

Part2.VI has two analysis modes (see ANALYSIS TYPE on the panel). The first mode is FRF, and the second mode is MODE SHAPE. In the FRF mode, the VI program computes FRFs based on input and output signals.

- 1) **Set Analysis Type**– to “FRF”
- 2) **Set Path** – Set files which stored the physical input and output data in “Path” .
- 3) **Average Parameter**– Users can set the Average Mode, the Number of Average, Weighting Mode, and Linear Mode.
- 4) **FRF Parameter**– Users can set the Window, Block Size, Sampling Rate and FRF Mode.
- 5) **Restart**– Restarting average. Skip if it is the first run.
- 6) **Run**



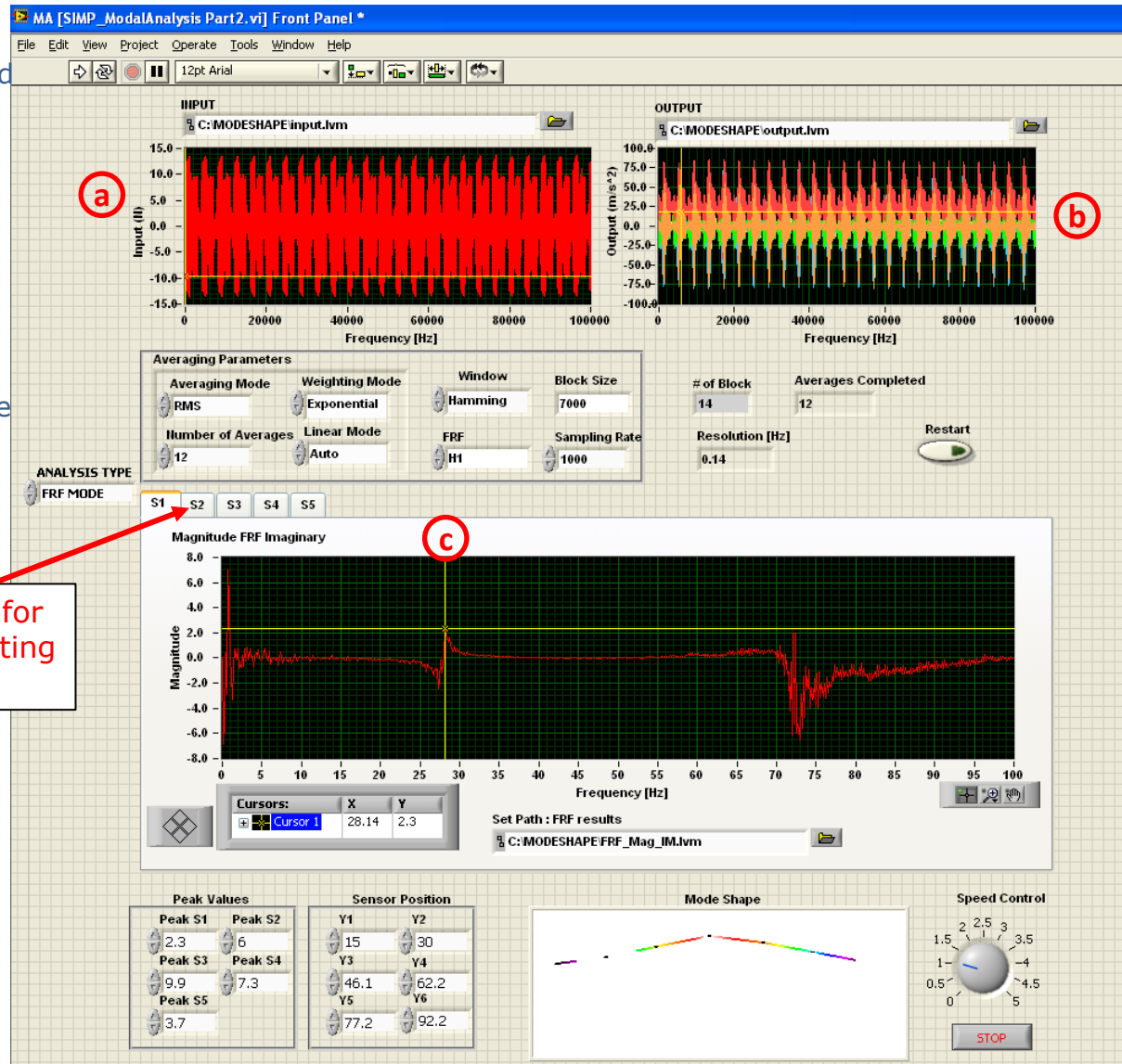
## PART 2.VI

After the first run, panel shows input and output signals and their FRF plots in the plot changer.

- a is a plot of input force data
- b are plots of output accelerations.
- c is a plot of imaginary part of FRF computed from input and output

By selecting different tabs, users can see other FRF graphs.

Tabs for selecting input



## FRF Parameters

☐ **averaging mode** specifies the averaging mode.

0	No averaging (default)
1	Vector averaging
2	RMS averaging

☐ **weighting mode** specifies the **weighting mode** for RMS and vector averaging.

0	Linear
1	Exponential (default)

☐ **number of averages** specifies the number of averages used for RMS and vector averaging.

☐ If **weighting mode** is exponential, the averaging process is continuous.

☐ If **weighting mode** is linear, the averaging process stops after this VI computes the selected **number of averages**.

☐ **linear mode** specifies the behavior of the averaging if you set the value of the **weighting mode** parameter to **Linear**. If you set **weighting mode** to **Exponential**, **linear mode** is ignored.

0	<b>One shot</b> —(default) Specifies that the averaging process stops once the value of <b>number of averages</b> is reached.
1	<b>Auto restart</b> —Specifies that the averaging process automatically restarts after the value of <b>number of averages</b> is reached.

## FRF Parameters

- ❑ **window** specifies the time-domain window to use. The default is Hanning.

0	Rectangle
1	Hanning (default)
2	Hamming
3	Blackman-Harris
4	Exact Blackman
5	Blackman
6	Flat Top
7	Four Term Blackman-Harris
8	Seven Term Blackman-Harris
9	Low Sidelobe
11	Blackman Nuttall
30	Triangle
60	Kaiser
61	Dolph-Chebyshev
62	Gaussian

- ❑ **block size** specifies the number of data contained in each block.

- ❑ **FRF mode** specifies how to compute the frequency response function (FRF). **FRF mode** determines whether H1, H2, or H3 is computed when performing frequency response measurements. The default is H1.

\* **Note FRF mode** only applies to RMS or vector averaging. **FRF mode** changes the result in RMS averaging but not in vector averaging. In vector averaging,  $H1 = H2 = H3$ .

0	<b>H1</b> (default)
1	<b>H2</b>
2	<b>H3</b>

## FRF Parameters

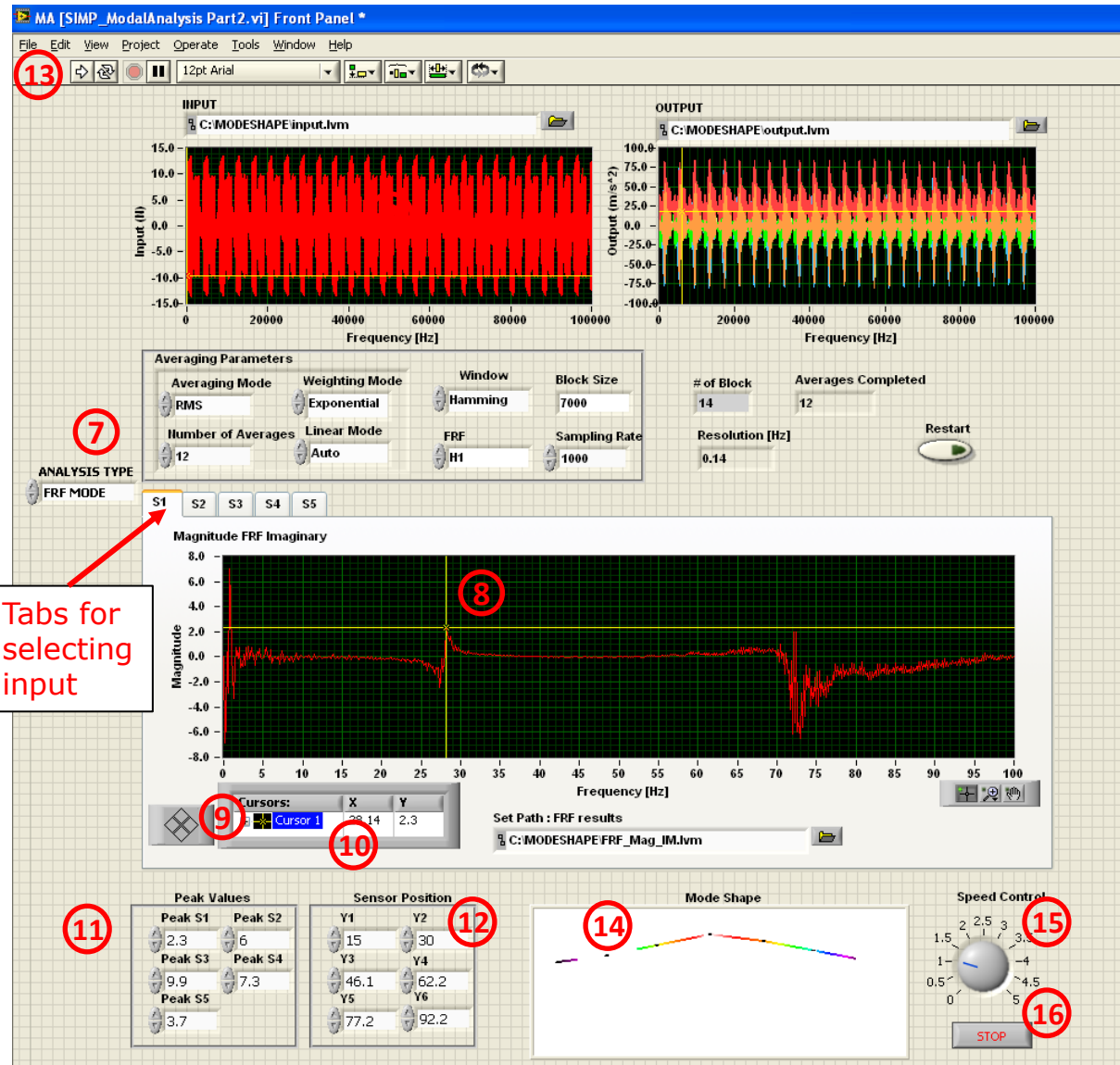
- ❑ **sampling rate** specifies the sampling rate of original input and output signal. For the FRF the sampling rate of input and output signal should be same.
- ❑ **# of block** specifies the number of blocks based on original signal and block size. The number of block is equivalent to the maximum number of average.
- ❑ **resolution** specifies the frequency resolution of FRF graph.
- ❑ **averages completed** returns the number of averages completed by the VI at that time.
- ❑ **averaging done** returns TRUE when **averages completed** is greater than or equal to the **number of averages** specified in **averaging parameters**. Otherwise, **averaging done** returns FALSE. **averaging done** is always TRUE if the selected **averaging mode** is No averaging.

Note : Users can find more details about FRF VI program at the website of [www.ni.com](http://www.ni.com)

## PART 2.VI

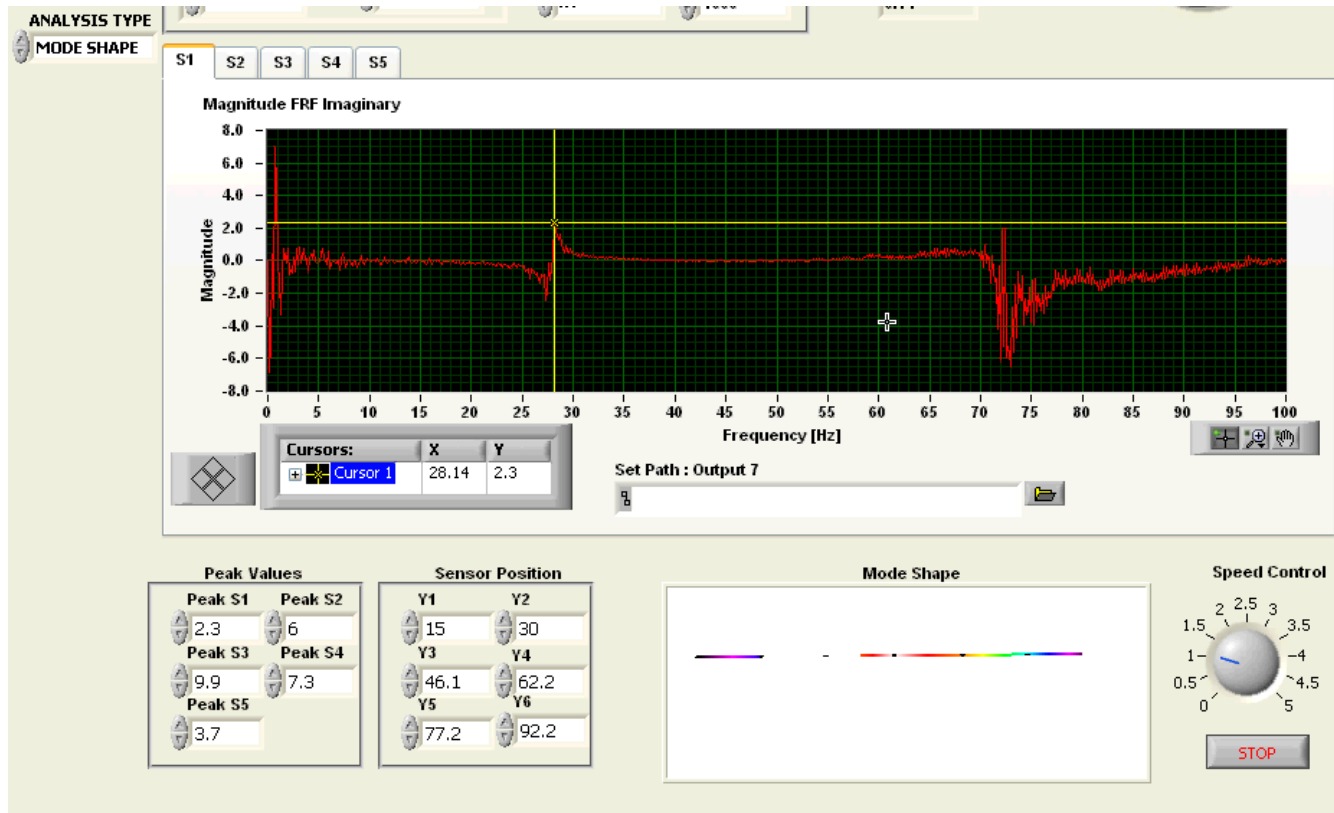
After FRF run, panel shows input and output signals and their FRFs in the tab. Based on the FRF results, the mode shape can be extracted.

- 7) **Set Analysis Type**– to “MODE SHPAE”
- 8) **Cursor** – users can read the value of the plot using cursor. The movement of cursor can be done using mouse click or cursor operation button.
- 9) **Cursor operation button**– users can move the cursor to find the peak value.
- 10) **Peak frequency**– users can read the peak values here.
- 11) **Peak Values**– five peak values can be obtained by repeating the step 8, step 9 and step 10 on the tab of interest.
- 12) **Sensor Position (cm)**– five 1D coordinates of sensors are from Y1 to Y5 and Y6 is the coordinate of end point of structure.
- 13) **Run**
- 14) **Mode Shape** – animation of the mode shape will be appear
- 15) **Speed Controller** – users can control the speed of the animation
- 16) **Stop**– stop the animation of mode shape

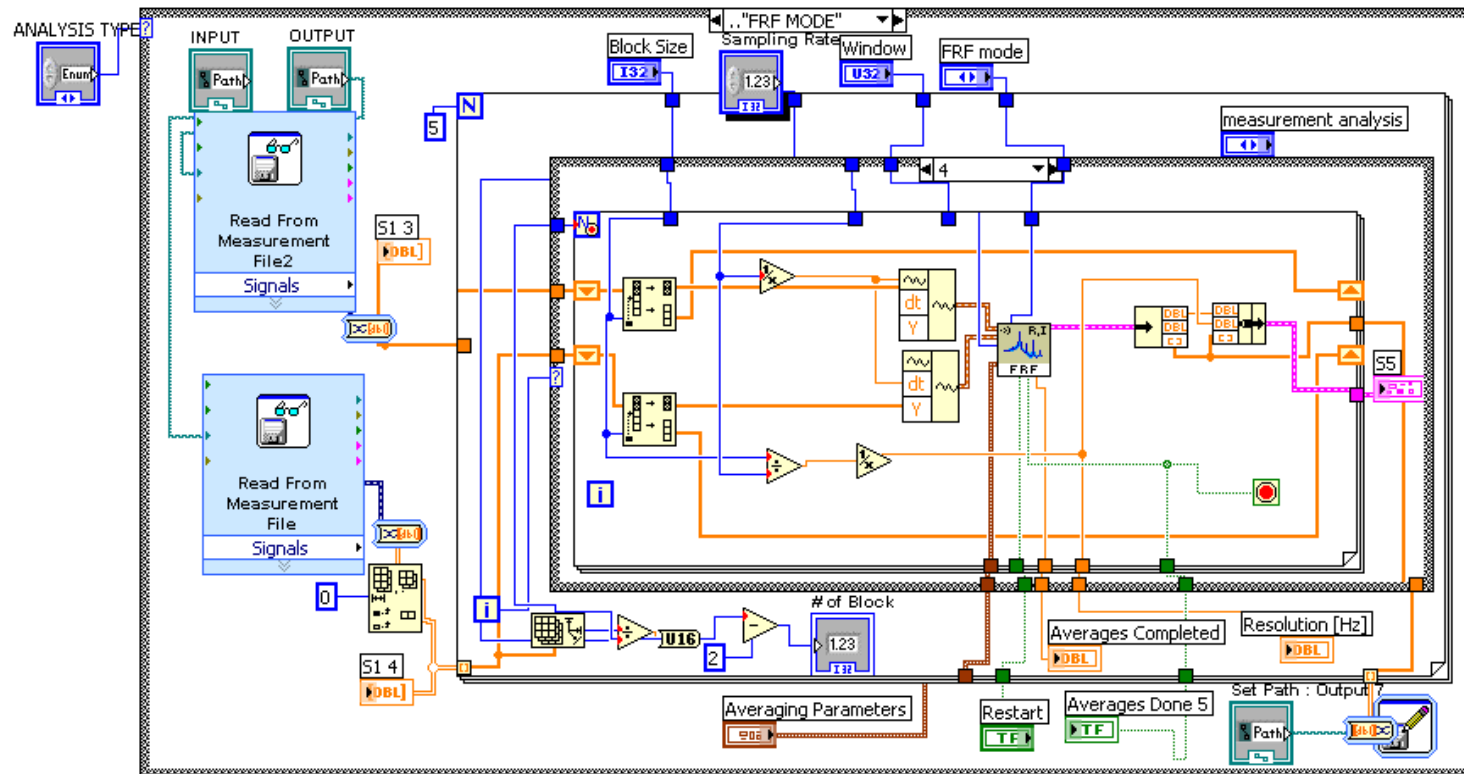




## Modal Analysis PART 2.VI

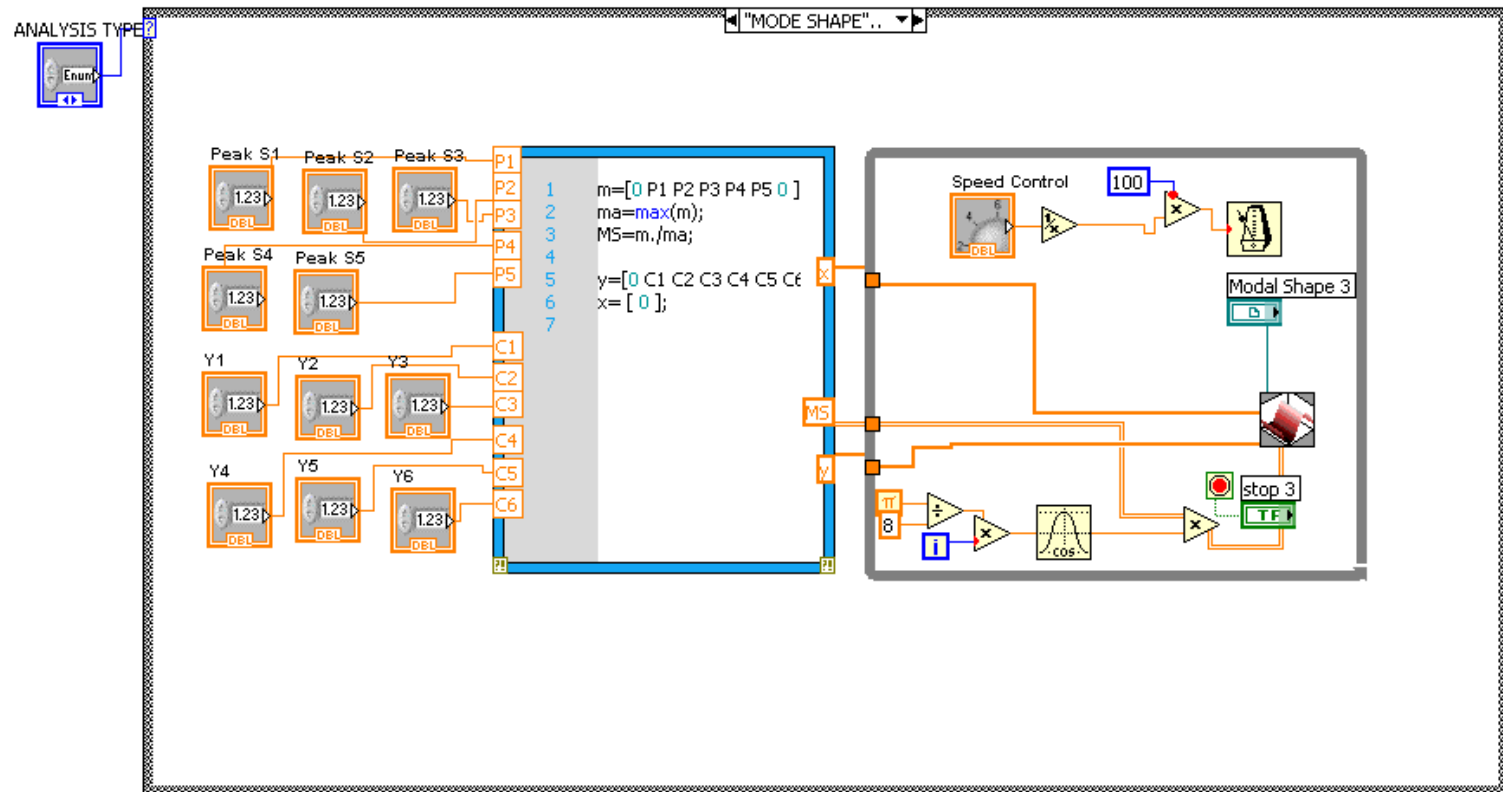


Captured video : The first mode shape of the tested beam



STEP1-The First Stack of Block Diagram of PART2.VI

- ❑ In STEP1 of the case structure, the VI program reads physical input excitation and output response signals and compute FRFs and display them.
- ❑ MATLAB script was used to divide the signals for the given FRF parameters (block size i.e. window size).



STEP2-The Second Stack of Block Diagram of PART2.VI

- ❑ In STEP2 of the case structure, the VI program reads the peak values which are selected and typed in by users and plots the mode shape
- ❑ MATLAB was used to assemble and normalize the mode shape.
- ❑ Animation of the moving mode shape will be presented.

# 10

## References

# References

---

- Roy R. Craig, Jr. and Andrew J. Kurdila, "Fundamentals of Structural Dynamics", 2<sup>nd</sup> Ed. Wiley